Solving Absolute Value Equations

Solving absolute value equations is almost the exact same as solving regular equations with one major difference. In most cases you have *2 solutions*.

Example:
$| x | = 5$

We know that when $x = 5$, $| 5 |$ will also equal 5, but it is also true that $| -5 |$ will equal 5. So, for $|x | = 5$, $x = \{-5, 5\}$. They both work.

How to solve absolute value equations

1) Isolate the absolute value.
2) Split into two separate equations, setting one to the negative and one to the positive.

Example:
$| 2x + 6 | - 3 = 13$

1) Isolate the absolute value:
** The steps are the same as if you were getting the x by itself. You move away all other numbers by doing the opposite operation:**

$$| 2x + 6 | - 3 = 13$$
$$+3 \quad +3$$
$$| 2x + 6 | = 16$$

2) Now split into two separate equations and solve each.

$$2x + 6 = -16 \qquad\qquad 2x + 6 = 16$$
$$-6 \quad -6 \qquad\qquad -6 \quad -6$$
$$\frac{2x}{2} = \frac{-22}{2} \qquad\qquad \frac{2x}{2} = \frac{10}{2}$$
$$x = -11 \qquad\qquad x = 5$$

3) Check by substituting in the original equation.

1) $| 6x + 12 | = 24$

2) $| 6 – 2x | = 14$

3) $|8x - 2| = 42$

4) $|\frac{2}{3}x + 6| = 2$

5) $|10 - \frac{3}{4}x| = 16$

6) $|4x - 12| = -36$

On #'s 1-10, notice how the steps of isolating the absolute value are the same as if you were isolating the x.

1) $5x + 9 = 144$

2) $5|\,3x - 6\,| + 9 = 144$

3) $\frac{x}{7} - 3 = 1$

4) $\frac{|\,12x - 8\,|}{7} - 3 = 1$

5) $\frac{2}{3}x - 11 = -3$

6) $\frac{2}{3}|2x - 10| - 11 = -3$

7) $\frac{4x - 5}{3} = 9$

8) $\frac{4|8x - 16| - 5}{3} = 9$

9) $\frac{5x + 7}{11} - 8 = -6$

10) $\frac{5|6x - 15| + 7}{11} - 8 = -6$

11) $|4x - 5| + 15 = 36$

12) $6|3x - 12| - 5 = 49$

13) $\frac{5}{8}|2x - 4| + 4 = -9$

14) $\frac{4|8x - 16| - 5}{7} = 9$

Practice Problems:

1) $4|6x - 12| + 9 = 129$

2) $\frac{1}{2}|8x + 4| - 7 = 27$

3) $-7|16x - 8| + 35 = -245$

4) $\frac{2|6x - 9|}{5} + 34 = 4$

5) $\frac{|4x - 24|}{9} = 8$

6) $\frac{3|7x + 28|}{8} - 4 = 17$

7) $\frac{3|15 - 5x| + 12}{13} = 9$

8) $\frac{5|11x + 33| - 12}{4} = 52$

9) $-2|\frac{1}{2}x + 8| - 5 = -25$

10) $6|\frac{1}{4}x - 4| - 73 = -13$

11) $4|5x - 10| + 52 = 12$

12) $\frac{3|12x - 36|}{8} - 7 = 11$

13) $\frac{3|4x + 32| + 4}{10} = 16$

14) $\frac{8|9x - 9| + 3}{15} = 5$

15) $\frac{2|35 - 7x|}{14} - 26 = -16$

16) $\frac{6|2x - 14| - 12}{4} + 11 = 53$

17) $-2|\frac{1}{3}x - 12| - 5 = -53$

18) $\frac{2}{3}|12 - \frac{1}{2}x| - 13 = 5$

19) $4|3x + 18| + 140 = 8$

20) $\frac{|16x - 4|}{9} - 7 = -3$

Answer Key:

1) x = {-3, 7}	2) x = {-9,8}	3) x = {-2,3}	4) No Solution
5) x = {-12, 24}	6) x = {-12,4}	7) x ={-4, 10}	8) x = {-7,1}
9) x = {-36, 4}	10) x ={-24, 56}	11) No Solution	12) x ={-1,7}
13) x ={-21, 5}	14) x ={0,2}	15) x ={-5, 15}	16) x = {-8, 22}
17) x = {-36,108}	18) x = {-30, 78}	19) No Solution	20) x = {-2 ,2.5}

Q1 Quiz 6 Review

1) $4|3x - 12| + 5 = 65$

2) $6\ |10x + 25| - 7 = 143$

3) $2|16x - 48| + 38 = 6$

4) $\frac{2|15x - 30|}{5} - 34 = -4$

5) $\frac{|8x + 2|}{-11} = -6$

6) $\frac{4|8x + 12|}{5} - 18 = -2$

7) $\frac{3|7x - 35| - 1}{2} = 10$

8) $\frac{5|8x - 4| - 8}{4} = 63$

9) $-2|6x + 18| - 5 = -29$

10) $\frac{2}{3}|6x + 12| - 21 = -5$

11) $4|5x - 10| + 174 = 14$

12) $\frac{3|11x + 33|}{8} - 7 = 26$

13) $\frac{3|10 - 2x| + 2}{5} = 16$

14) $\frac{5|15 - 5x| + 8}{4} = 52$

15) $\frac{2|6x - 24|}{-9} + 5 = 17$

16) $\frac{6|6x - 6| + 4}{5} + 12 = 56$

Answer Key:

1) x = {-1,9}	2) x = {-5,0}	3) No Solution	4) x = {-3,7}
5) x = {-8.5,8}	6) x = {-4,1}	7) x ={4,6}	8) x = {-6,7}
9) x = {-5,-1}	10) x ={-6,2}	11) No Solution	12) x ={-11,5}
13) x ={-18,-8}	14) x ={-5,11}	15) No Solution	16) x = {-5,7}