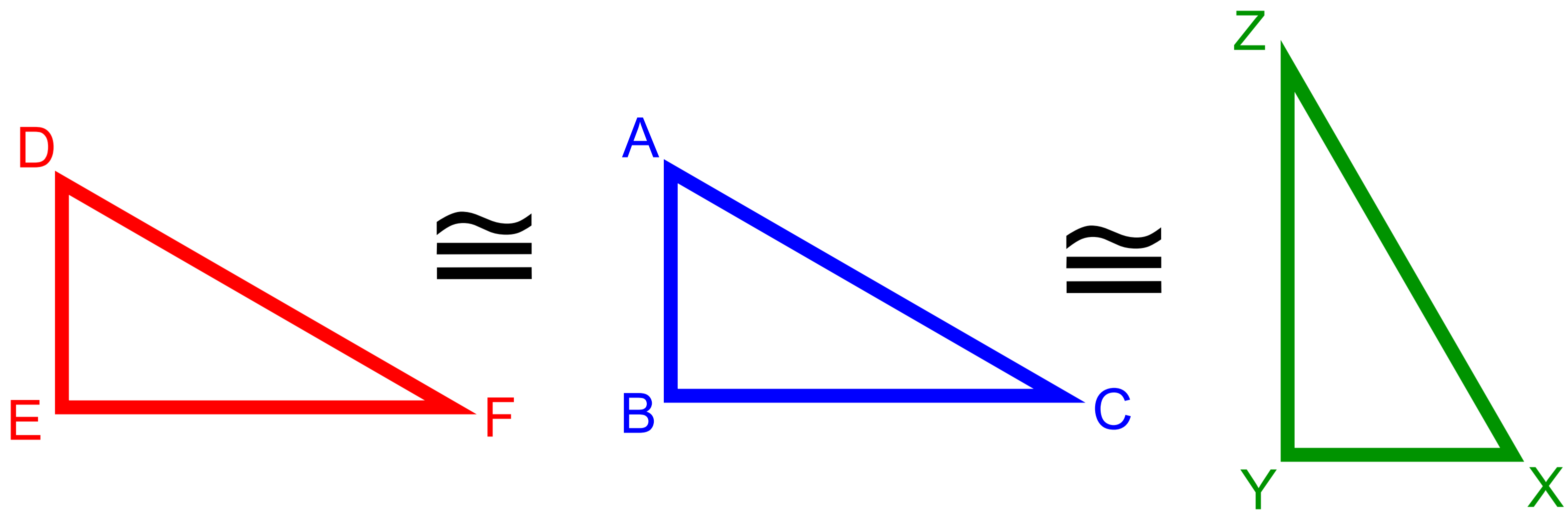
Congruent Triangles

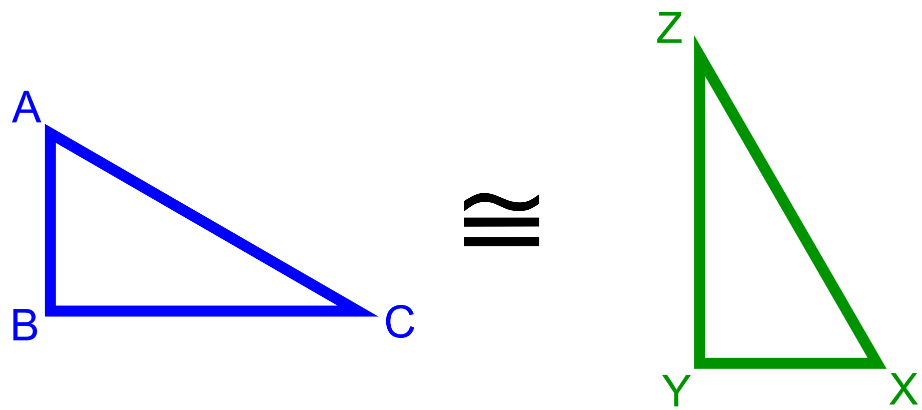
Two triangles are congruent if all of their corresponding parts are congruent. This means that congruent triangles will have three pairs of congruent sides and three pairs of congruent angles.

Congruent triangles must have the same size and shape, but may be positioned differently. Thus, in the figures below, $\Delta ABC \cong \Delta DEF$, and also $\Delta ABC \cong \Delta XYZ$.



When we say two triangles are congruent, the way in which we name the triangles is important! Vertices written in the same position correspond to one another, and are congruent. (So if $\Delta ABC \cong \Delta XYZ$, then angles A and X are congruent.) The sides between corresponding vertices are also congruent, so $XZ \cong AC$.

$$\Delta ABC \cong \Delta XYZ$$



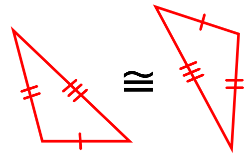
Corresponding Angles	**Corresponding Sides**
$\angle A \cong \angle X$	$AB \cong XY$
$\angle B \cong \angle Y$	$BC \cong YZ$
$\angle C \cong \angle Z$	$AC \cong XZ$

To determine whether or not two triangles are congruent, we do not need to know that ALL of the sides and ALL of the angles are congruent. We can actually know as few as three pairs of corresponding parts, as long as they fall into a specific pattern.

These are the ***only* ways to prove two triangles congruent!**

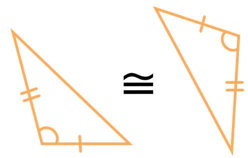
Side - Side - Side (SSS)

If three sides of one triangle are congruent to the corresponding three sides of another triangle, the two triangles are congruent.



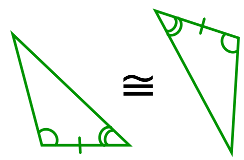
Side - Angle - Side (SAS)

If two sides and the ***included*** angle of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent.



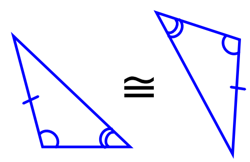
Angle - Side - Angle (ASA)

If two angles and the included side of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent.



Angle - Angle - Side (AAS)

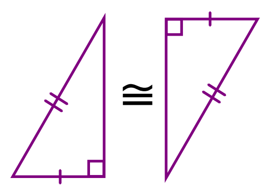
If two angles and a ***nonincluded*** side of one triangle are congruent to the corresponding parts of another triangle, the two triangles are congruent.



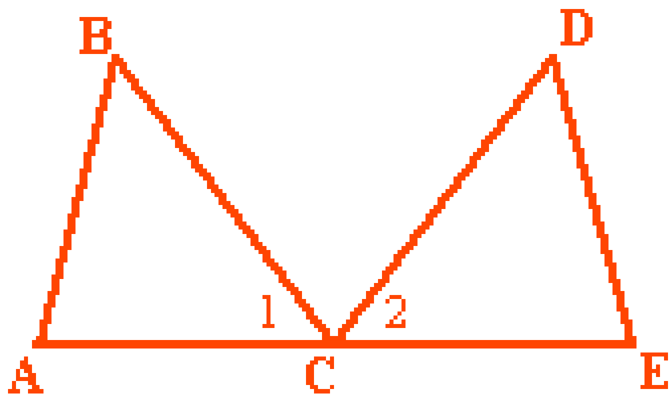
Hypotenuse - Leg (HL)

If the hypotenuse and a leg of one ***right*** triangle are congruent to the corresponding parts of another ***right*** triangle, the two triangles are congruent.

**This only works in right triangles!



Using the given information, which method would you use to prove the two triangles congruent?



Given: ΔABC, ΔEDC

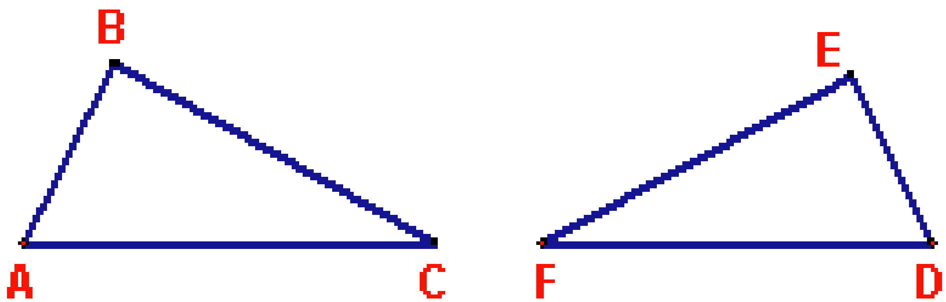
$\measuredangle 1 \cong \measuredangle 2$, $\measuredangle A \cong \measuredangle E$

and $\overline{AC} \cong \overline{EC}$

Prove: $\Delta ABC \cong \Delta EDC$

Answer: ASA
(AC is included between $\angle$A and $\angle$1)

Using the given information, which method would you use to prove the two triangles congruent?



Given: $\overline{AB} \cong \overline{DE}$

$\angle B \cong \angle E$

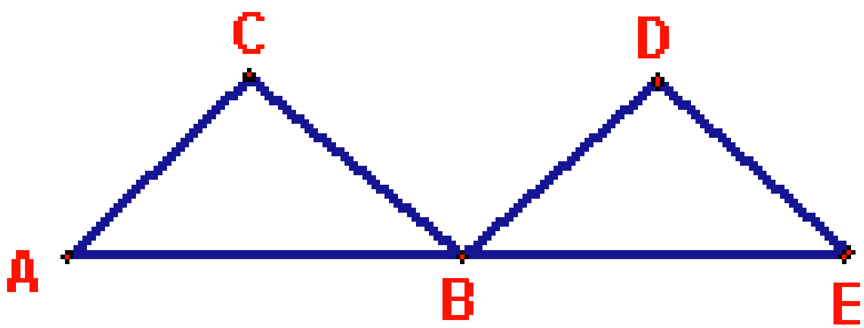
$\overline{BC} \cong \overline{EF}$

Prove: $\Delta ABC \cong \Delta DEF$

Answer: SAS

($\angle$B is included between AB and BC)

Using the given information, which method would you use to prove the two triangles congruent?



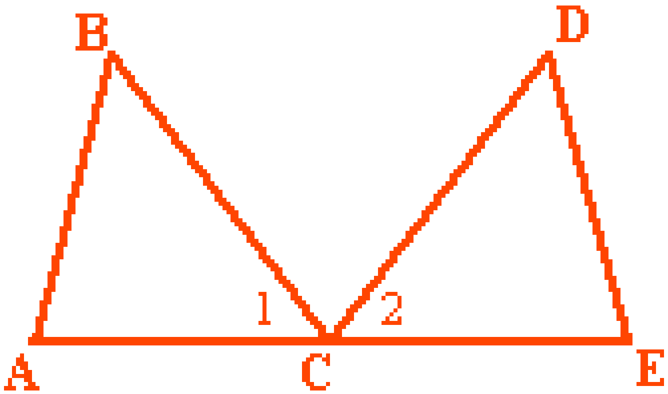
Given: $\measuredangle A \cong \measuredangle E$

$\measuredangle CBA \cong \measuredangle DBE$

B is midpoint of $\overline{AE}$

Prove: $\Delta ABC \cong \Delta EBD$

Answer: ASA



Given: <1 ≅ <2
BC ≅ DC

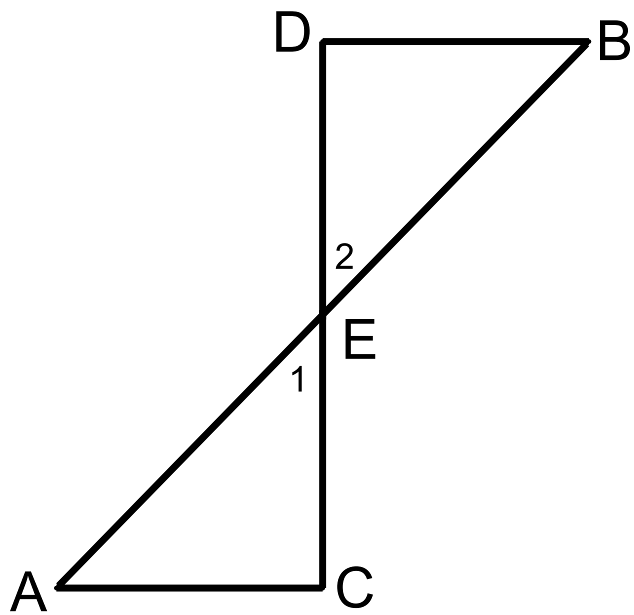
Prove: ΔABC ≅ ΔEDC

What additional information would I need to prove that ΔABC ≅ ΔEDC by:

ASA: <B ≅ <D

SAS: AC ≅ EC

SAA: <A ≅ <E



Given: $\angle C \cong \angle D$

$DE \cong CE$

Prove: $\Delta ACE \cong \Delta BDE$

What additional information would I need to prove that $\Delta ABC \cong \Delta BDE$ by:

ASA: <1 ≅ <2 ← We know this! (vertical angles)

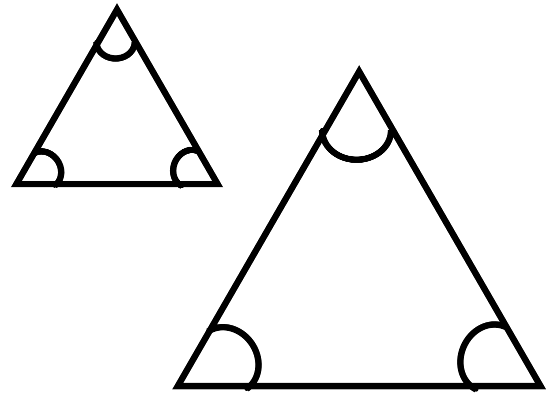
SAS: AC ≅ BD

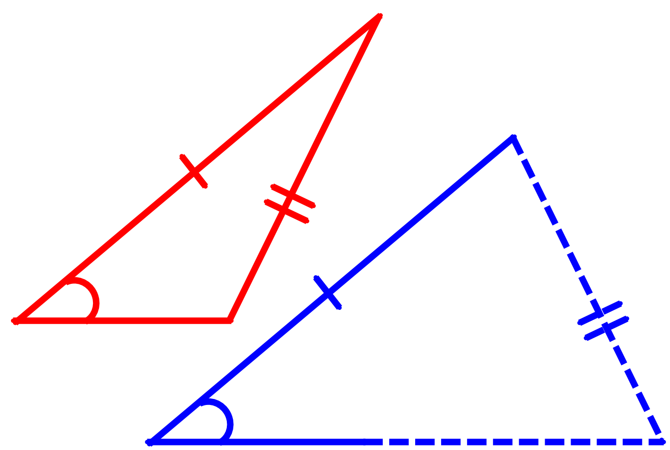
SAA: <A ≅ <B

Ways that WON'T work to prove congruence:

Angle - Angle - Angle

Two triangles that have all angles congruent will be ***similar*** (their sides will be proportional), but not necessarily congruent.





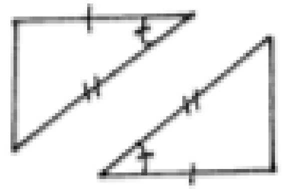
Angle - Side - Side

There are actually ***two*** possible triangles that can be constructed from two given sides and a nonincluded angle.

Practice Problems!

Determine which reason, ***if any***, can be used to prove that the two triangles are congruent. (If congruence cannot be proven, write *none*.)

1)



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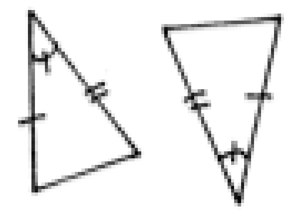

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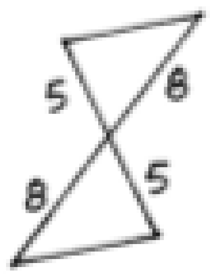

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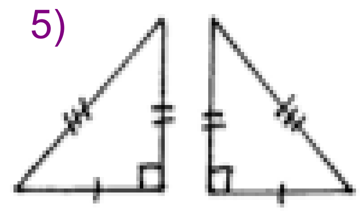
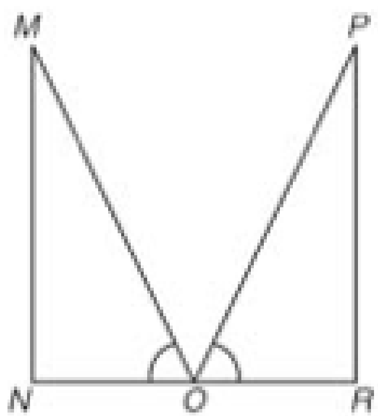
5)



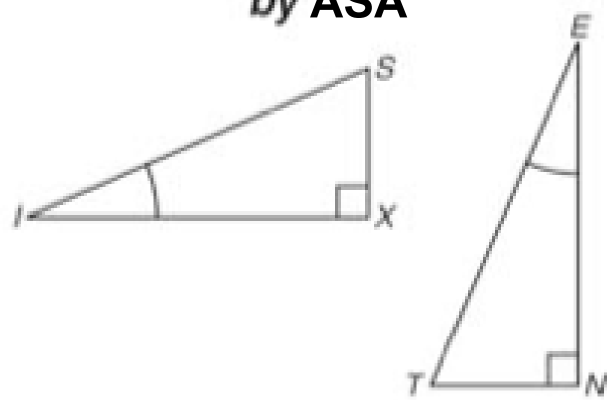
Practice Problems!

What extra information do I need to prove:

6) $\triangle MNO \cong \triangle PRO$ *by SAS*

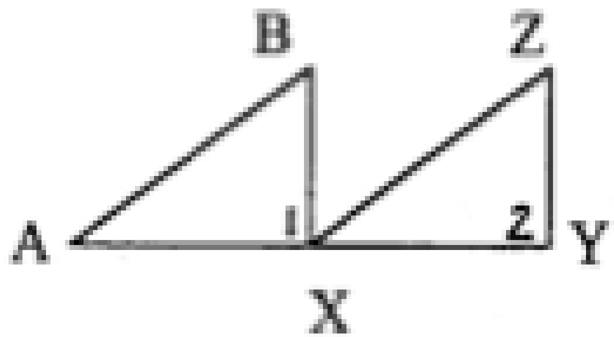


7) $\triangle SIX \cong \triangle TEN$ *by* **ASA**



Practice Problems!

8)



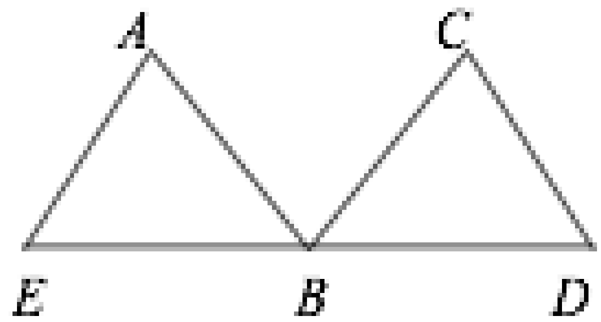
If <B ≅ <Z, AX ≅ XY, and <1 ≅ <2,

Write a congruence statement: $\Delta ABX \cong \Delta XZY$

Why are these triangles congruent? SAA

Practice Problems!

9)



If AE ≅ CB, AB ≅ CD, and B is the **midpoint** of ED,

Write a congruence statement: $\Delta AEB \cong \Delta CBD$

Why are these triangles congruent? SSS