

Solving Linear Equations

Golden Rule of Algebra:

“Do unto one side of the equal sign as you will do to the other...”

Whatever you do on one side of the equal sign, you MUST do the same exact thing on the other side. If you multiply by -2 on the left side, you have to multiply by -2 on the other. If you subtract 15 from one side, you must subtract 15 from the other. You can do whatever you want (to get the x by itself) as long as you do it on both sides of the equal sign.

Solving Single Step Equations:

To solve single step equations, you do the *opposite* of whatever the operation is. The opposite of addition is subtraction and the opposite of multiplication is division.

Solve for x:

1) $x + 5 = 12$

2) $x - 11 = 19$

3) $22 - x = 17$

4) $5x = -30$

5) $\frac{x}{-5} = 3$

6) $\frac{2}{3}x = -8$

Solving Multi-Step Equations:

$3x - 5 = 22$ To get the x by itself, you will need to get rid of the 5 and the 3.

$\frac{+5}{-5}$ We do this by going in opposite order of PEMDAS. We get rid of addition and subtraction first.

$\frac{3x}{3} = \frac{27}{3}$ Then, we get rid of multiplication and division.

$$x = 9$$

We check the answer by putting it back in the original equation:

$$3x - 5 = 22, x = 9$$

$$3(9) - 5 = 22$$

$$27 - 5 = 22$$

$$22 = 22 \text{ (It checks)}$$

Simple Equations:

$$1) 9x - 11 = -38$$

$$2) 160 = 7x + 6$$

$$3) 32 - 6x = 53$$

$$4) -4 = 42 - 4x$$

$$5) \frac{3}{4}x - 11 = 16$$

$$6) 37 = 25 - (\frac{2}{3})x$$

7) $4x - 7 = -23$

8) $12x + 9 = -15$

9) $21 - 4x = 45$

10) $(x/7) - 4 = 4$

11) $(-x/5) + 3 = 7$

12) $26 = 60 - 2x$

Equations with more than 1 x on the same side of the equal sign:

You need to simplify (combine like terms) and then use the same steps as a multi-step equation.

Example:

$9x - 5x = 4x$ and
 $11 + 10 = 21$

$9x + 11 - 5x + 10 = -15$

$4x + 21 = -15$

$\underline{-21} \quad \underline{-21}$

$\frac{4x}{4} = \frac{-36}{4}$

$x = -9$

Now it looks like a multistep eq. that we did in the 1st

Use subtraction to get rid of the addition.

Now divide to get rid of the multiplication

13) $15x - 24 - 4x = -79$

14) $102 = 69 - 7x + 3x$

15) $3(2x - 5) - 4x = 33$

16) $3(4x - 5) + 2(11 - 2x) = 43$

17) $9(3x + 6) - 6(7x - 3) = 12$

18) $7(4x - 5) - 4(6x + 5) = -91$

19) $8(4x + 2) + 5(3x - 7) = 122$

Equations with x's on BOTH sides of the equal sign:

You need to "Get the X's on one side and the numbers on the other." Then you can solve.

Example: $12x - 11 = 7x + 9$

$$\begin{array}{r} -7x \quad -7x \\ \hline 5x - 11 = 9 \\ +11 \quad +11 \\ \hline 5x \quad = 20 \\ 5 \quad 5 \end{array}$$

$$x = 4$$

Move the x's to one side.

Now it looks like a multistep equation that we did in the 1st section.

Add to get rid of the subtraction.

Now divide to get rid of the multiplication

20) $11x - 3 = 7x + 25$

21) $22 - 4x = 12x + 126$

23) $\frac{3}{4}x - 12 = \frac{1}{2}x - 6$

24) $5(2x + 4) = 4(3x + 7)$

25) $12(3x + 4) = 6(7x + 2)$

26) $3x - 25 = 11x - 5 + 2x$

Q3 Quiz 2 Review

Part I:

1) $\frac{5}{8}x + 11 = -4$

2) $45 - \frac{3}{4}x = -9$

3) $\frac{2}{3}x - 120 = 80$

4) $54 - \frac{3}{8}x = 63$

5) $7(4x + 3) - 8(3x - 2) = -3$

6) $6(5x - 1) = 4(10x + 16)$

7) $9(6x + 3) + 8(2x - 11) = -271$

8) $6(6x + 1) = 19(3x - 3)$

9) $3(8x + 6) = 8(4x + 2)$

10) $5(11x - 5) - 7(9x - 2) = 37$

11) $10(5x + 15) = 7(8x + 12)$

12) $4(8x + 3) - 6(7x - 8) = 35$

Answer Key:

1) $x = -24$

2) $x = 72$

3) $x = 300$

4) $x = -24$

5) $x = -10$

6) $x = -7$

7) $x = -3$

8) $x = 3$

9) $x = \frac{1}{4}$

10) $x = -6$

11) $x = 11$

12) $x = 2.5$

Part II:

Show all work:

1) $\frac{3}{8}x + 91 = 67$

2) $43 - \frac{2}{3}x = 81$

3) $\frac{3}{4}x - 84 = 9$

4) $54 - \frac{5}{8}x = -1$

5) $6(4x + 3) - 8(4x - 2) = 106$

6) $3(4x - 5) = 5(2x - 1)$

7) $8(8x + 3) + 9(4x - 2) = 306$

8) $5(6x - 5) = 19(2x - 3)$

9) $3(10x - 7) = 11(2x + 1)$

10) $8(11x - 5) - 9(10x - 2) = 8$

11) $6(3x + 5) = 7(2x + 2)$

12) $9(8x + 3) - 8(6x - 2) = -5$

Answer Key:

- 1) $x = -64$ 2) $x = -57$ 3) $x = 124$ 4) $x = 88$ 5) $x = -9$ 6) $x = 5$
7) $x = 3$ 8) $x = 4$ 9) $x = 4$ 10) $x = -15$ 11) $x = -4$ 12) $x = -2$

Solving Quadratic Equations

Solving quadratic equations (equations with x^2 can be done in different ways. We will use two different methods. What both methods have in common is that the equation has to be set to = 0. For instance, if the equation was $x^2 - 22 = 9x$, you would have to subtract $9x$ from both sides of the equal sign so the equation would be $x^2 - 9x - 22 = 0$.

Solve by factoring: After the equation is set equal to 0, you factor the trinomial.

$$x^2 - 9x - 22 = 0$$

$$(x-11)(x+2) = 0$$

Now you would set each factor equal to zero and solve. Think about it, if the product of the two binomials equals zero, well then one of the factors has to be zero.

$$x^2 - 9x - 22 = 0$$

$$(x-11)(x+2) = 0$$

$$\begin{array}{r} x - 11 = 0 \quad x + 2 = 0 \\ \underline{+11 \quad +11} \quad \underline{-2 \quad -2} \end{array}$$

$$\begin{array}{l} x = 11 \quad \text{or} \quad x = -2 \\ x = \{-2, 11\} \end{array}$$

* Check in the ORIGINAL equation!

Solving each quadratic by factoring:

$$1) x^2 - 5x - 14 = 0$$

$$2) x^2 + 11x = -30$$

$$3) x^2 - 45 = 4x$$

Equations Packet

Packet #5

4) $x^2 = 15x - 56$

5) $3x^2 + 9x = 54$

6) $x^3 = x^2 + 12x$

7) $25x^2 = 5x^3 + 30x$

8) $108x = 12x^2 + 216$

9) $3x^2 - 2x - 8 = 2x^2$

10) $10x^2 - 5x + 11 = 9x^2 + x + 83$

11) $4x^2 + 3x - 12 = 6x^2 - 7x - 60$

12) $6x^2 - 5x - 11 = 7x^2 - 5x - 47$

13) $4x^2 - 9x + 3 = 2x^2 + 5x + 3$

Q3 Quiz 3 Review

1) $x^2 = 22x - 96$

2) $5x^2 - 3x - 15 = 6x^2 + 10x + 15$

3) $8x^2 - 6x + 72 = 9x^2 - 6x - 72$

4) $5x = x^2 - 84$

5) $6x^2 = 30x + 396$

6) $x^3 + 39x = 16x^2$

7) $20x^2 - 45 = 0$

8) $3x^2 = -3x + 216$

9) $5x^3 - 221x = 5x^2$

10) $5x^2 + 35x = 40$

11) $21x^2 + 3x - 10 = 5x^2 + 3x - 9$

12) $x^2 = -22x - 72$

13) $11x^2 - 4x - 15 = 3x^2 - 4x - 4$

14) $4x^2 - 17x + 3 = 6x^2 - 9x + 3$

15) $8x^2 + 2x - 11 = 5x^2 + 9x - 11$

16) $3x^2 - 6x + 8 = 12x^2 + 21x + 8$

Answer Key:

1) $x = \{6, 16\}$

2) $x = \{-10, -3\}$

3) $x = \{-12, 12\}$

4) $x = \{-7, 12\}$

5) $x = \{-6, 11\}$

6) $x = \{0, 3, 13\}$

7) $x = \{-\frac{3}{2}, \frac{3}{2}\}$

8) $x = \{-9, 8\}$

9) $x = \{-6, 0, 7\}$

10) $x = \{-8, 1\}$

11) $x = \{-\frac{1}{4}, \frac{1}{4}\}$

12) $x = \{-18, -4\}$

13) $x = \{-\frac{1}{2}, \frac{1}{2}\}$

14) $x = \{-2, 0\}$

15) $x = \{0, 3\}$

16) $x = \{-3, 0\}$

Solve using the quadratic formula:

Solve $x^2 - 9x - 22 = 0$ using the *quadratic formula*

When $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a is the coefficient of x^2 **b** is the coefficient of x **c** is the number (third term)

Notice the \pm is what will give your two answers (just like you had when solving by factoring)

$$x^2 - 9x - 22 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = -9$$

$$c = 22$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(22)}}{2(1)} \rightarrow (-9)^2 - 4(1)(22) \text{ can be done in one step in the calculator (leave out radical!!).}$$

$$x = \frac{9 \pm \sqrt{169}}{2}$$

Split and do the + side and - side

$$\frac{9 - 13}{2} \qquad \frac{9 + 13}{2}$$

$$x = -2 \qquad \text{or} \qquad x = 11$$

$$x = \{-2, 11\}$$

* *Check in the ORIGINAL equation!*

Remember, ALL SUBSTITUTIONS must be done in PARENTHESES!!!!!!!

Solving each quadratic using the Quadratic Formula:

14) $2x^2 - 6x + 1 = 0$

15) $3x^2 + 2x = 3$

16) $4x^2 + 2 = -7x$

17) $7x^2 = 3x + 2$

18) $3x^2 + 6 = 5x$

19) $9x - 3 = 4x^2$

20) $4x^2 - 5x = 2$

21) $11x^2 - 3 = -4x$

Answer Key:

14) $x = \frac{3}{2} \pm \frac{\sqrt{7}}{2}$

15) $x = \frac{-1}{3} \pm \frac{\sqrt{10}}{3}$

16) $x = \frac{-7}{8} \pm \frac{\sqrt{17}}{8}$

17) $x = \frac{3}{14} \pm \frac{\sqrt{65}}{14}$

18) No Real Solution

19) $x = \frac{9}{8} \pm \frac{\sqrt{33}}{8}$

20) $x = \frac{5}{8} \pm \frac{\sqrt{57}}{8}$

21) $x = \frac{-2}{11} \pm \frac{\sqrt{37}}{11}$

Q3 Quiz 4 Review

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You must write the quadratic formula for each problem

1) $5x^2 - 3 = -4x$

2) $6x^2 = 12x - 5$

3) $2x^2 - 5x = 4$

4) $x^2 = -14x - 19$

5) $4x^2 = 6x + 3$

6) $7x^2 + x + 3 = 0$

7) $16x^2 + 1 = 12x$

8) $6x^2 = 7x + 1$

9) $12x^2 - 13x - 4 = 0$

10) $8x^2 + 16x + 3 = 0$

Answer Key:

1) $x = \frac{-2}{5} \pm \frac{\sqrt{19}}{5}$ 2) $x = 1 \pm \frac{\sqrt{6}}{6}$ 3) $x = \frac{5}{4} \pm \frac{\sqrt{57}}{4}$ 4) $x = -7 \pm \sqrt{30}$

5) $x = \frac{3}{4} \pm \frac{\sqrt{21}}{4}$ 6) *No Real Solution* ($\sqrt{-83}$) 7) $x = \frac{3}{8} \pm \frac{\sqrt{5}}{8}$

8) $x = \frac{7}{12} \pm \frac{\sqrt{73}}{12}$ 9) $x = \{-\frac{1}{4}, \frac{4}{3}\}$ 10) $x = -1 \pm \frac{\sqrt{10}}{4}$

Factor:

1) $x^2 + 4x + 4$

2) $x^2 - 6x + 9$

3) $x^2 - 18x + 81$

4) $x^2 + 10x + 25$

5) $x^2 - 20x + 100$

6) $x^2 + 8x + 16$

7) $x^2 - 22x + 121$

8) $x^2 + 32x + 256$

9) $x^2 - 40x + 400$

Completing the Square

Completing the square is another method that is used to solve quadratic equations. This method is especially helpful when the quadratic equation cannot be solved by simply factoring.

Remember the standard form for a quadratic equation is: $ax^2 + bx + c = 0$.

Example:

1. $x^2 + 8x - 9 = 0$

$$x^2 + 8x - 9 = 0$$

$$\quad +9 \quad +9$$

$$x^2 + 8x = 9$$

$$\left(\frac{1}{2}(8)\right)^2 = (4)^2 = 16$$

$$x^2 + 8x + 16 = 9 + 16$$

$$x^2 + 8x + 16 = 25$$

$$(x + 4)(x + 4) = 25$$

$$(x + 4)^2 = 25$$

$$\sqrt{(x + 4)^2} = \sqrt{25}$$

$$x + 4 = \pm 5$$

$$x + 4 = 5 \qquad x + 4 = -5$$

$$\quad -4 \quad -4 \qquad \quad -4 \quad -4$$

$$x = 1 \qquad \qquad \qquad x = -9$$

$$x = \{-9, 1\}$$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.
4. To create the perfect square trinomial:
 - a) Take $\left(\frac{1}{2}b\right)^2$
 - b) Add that value to both sides of the equation.
5. Factor the perfect square trinomial.
6. Rewrite the factors as a squared binomial.
7. Take the square root of both sides.
8. Split the solution into two equations
9. Solve for x.
10. Create your final answer.

More Examples:

1) $x^2 + 2x - 3 = 0$

2) $x^2 - 16x + 60 = 0$

3) $x^2 - 8x + 7 = 0$

Example:

1. $x^2 - 10x - 7 = 0$

$$x^2 - 10x - 9 = 0$$

$$\quad +7 \quad +7$$

$$x^2 - 10x = 7$$

$$\left(\frac{1}{2}(-10)\right)^2 = (-5)^2 = 25$$

$$x^2 + 10x + 25 = 7 + 25$$

$$x^2 + 10x + 25 = 32$$

$$(x - 5)(x - 5) = 32$$

$$(x - 5)^2 = 32$$

$$\sqrt{(x - 5)^2} = \sqrt{32}$$

$$x - 5 = \pm 4\sqrt{2}$$

$$\quad +5 \quad +5$$

$$x = 5 \pm 4\sqrt{2}$$

Steps:

1. Be sure that the coefficient of the highest exponent is 1. If it is not divide each term by that value to create a leading coefficient of 1.
2. Move the constant term to the right hand side.
3. Prepare to add the needed value to create a perfect square trinomial. Be sure to balance the equation.
4. To create the perfect square trinomial:
 - a) Take $\left(\frac{1}{2}b\right)^2$
 - b) Add that value to both sides of the equation.
5. Factor the perfect square trinomial.
6. Rewrite the factors as a squared binomial.
7. Take the square root of both sides.
8. **Isolate X. Since you cannot combine it with $\pm 4\sqrt{2}$, you do not need to split it into two equations.**
9. Create your final answer

Equations Packet

4) $x^2 + 12x + 4 = 0$

5) $x^2 - 8x - 11 = 0$

6) $x^2 + 6x - 36 = 0$

7) $x^2 + 4x - 44 = 0$

8) $x^2 + 24x + 24 = 0$

9) $x^2 - 16x - 6 = 0$

Packet #5

Equations Packet

Packet #5

10) $x^2 + 6x + 4 = 0$

11) $x^2 - 10x + 5 = 0$

12) $x^2 + 2x - 47 = 0$

13) $x^2 + 4x - 92 = 0$

14) $x^2 - 28x + 4 = 0$

15) $x^2 - 12x - 20 = 0$

Equations Packet

Packet #5

16) $x^2 + 18x + 1 = 0$

17) $x^2 - 22x - 39 = 0$

18) $x^2 + 8x - 164 = 0$

Q3 Quiz 5 Review

Solve each quadratic using completing the square:

1) $x^2 + 6x - 112 = 0$

2) $x^2 - 20x + 64 = 0$

3) $x^2 - 10x + 7 = 0$

4) $x^2 + 8x + 8 = 0$

5) $x^2 - 2x - 23 = 0$

6) $x^2 + 6x - 63 = 0$

7) $x^2 + 20x - 8 = 0$

8) $x^2 - 24x + 16 = 0$

9) $x^2 - 18x - 88 = 0$

10) $x^2 + 12x - 39 = 0$

11) $x^2 + 4x - 60 = 0$

12) $x^2 - 14x + 17 = 0$

13) $x^2 - 10x + 7 = 0$

14) $x^2 + 8x + 15 = 0$

15) $x^2 - 2x - 80 = 0$

16) $x^2 + 6x - 39 = 0$

17) $x^2 + 20x - 25 = 0$

18) $x^2 - 24x + 23 = 0$

19) $x^2 - 18x - 40 = 0$

20) $x^2 + 12x - 18 = 0$

21) $x^2 + 15x + 26 = 0$

22) $x^2 - 10x - 25 = 0$

Answer Key:

1) $x = \{-14, 8\}$

2) $x = \{4, 16\}$

3) $x = 5 \pm 4\sqrt{2}$

4) $x = -4 \pm 2\sqrt{2}$

5) $x = 1 \pm 2\sqrt{6}$

6) $x = -3 \pm 6\sqrt{2}$

7) $x = -10 \pm 6\sqrt{3}$

8) $x = 12 \pm 8\sqrt{2}$

9) $x = \{-4, 22\}$

10) $x = -6 \pm 5\sqrt{3}$

11) $x = \{-10, 6\}$

12) $x = 7 \pm 4\sqrt{2}$

13) $x = 5 \pm 3\sqrt{2}$

14) $x = \{-5, -3\}$

15) $x = \{-8, 10\}$

16) $x = -3 \pm 4\sqrt{3}$

Solving Absolute Value Equations

Solving absolute value equations is almost the exact same as solving regular equations with one major difference. In most cases you have *2 solutions*.

Example:

$$|x| = 5$$

We know that when $x = 5$, $|5|$ will also equal 5, but it is also true that $|-5|$ will equal 5. So, for $|x| = 5$, $x = \{-5, 5\}$. **They both work.**

How to solve absolute value equations

- 1) Isolate the absolute value.
- 2) Split into two separate equations, setting one to the negative and one to the positive.

Example:

$$|2x + 6| - 3 = 13$$

- 1) Isolate the absolute value:

** The steps are the same as if you were getting the x by itself. You move away all other numbers by doing the opposite operation:**

$$\begin{aligned} |2x + 6| - 3 &= 13 \\ |2x + 6| \quad \underline{+3} \quad \underline{+3} &= 16 \end{aligned}$$

- 2) Now split into two separate equations and solve each.

$$\begin{aligned} 2x + 6 &= -16 \\ \underline{-6} \quad \underline{-6} & \\ \frac{2x}{2} &= \frac{-22}{2} \end{aligned}$$

$$x = -11$$

$$\begin{aligned} 2x + 6 &= 16 \\ \underline{-6} \quad \underline{-6} & \\ \frac{2x}{2} &= \frac{10}{2} \end{aligned}$$

$$x = 5$$

- 3) Check by substituting in the original equation

Solve each equation:

1) $|6x + 12| = 24$

2) $|6 - 2x| = 14$

3) $|8x - 2| = 42$

4) $|\frac{2}{3}x + 6| = 2$

5) $|10 - \frac{3}{4}x| = 16$

6) $|4x - 12| = -36$

On #'s 1-10, notice how the steps of isolating the absolute value are the same as if you were isolating the x.

1) $5x + 9 = 144$

2) $5|3x - 6| + 9 = 144$

3) $\frac{x}{7} - 3 = 1$

4) $\frac{|12x - 8|}{7} - 3 = 1$

5) $\frac{2}{3}x - 11 = -3$

6) $\frac{2}{3}|2x - 10| - 11 = -3$

7) $\frac{4x - 5}{3} = 9$

8) $\frac{4|8x - 16| - 5}{3} = 9$

$$9) \frac{5x + 7}{11} - 8 = -6$$

$$10) \frac{5|6x - 15| + 7}{11} - 8 = -6$$

$$11) |4x - 5| + 15 = 36$$

$$12) 6|3x - 12| - 5 = 49$$

$$13) \frac{5}{8}|2x - 4| + 4 = -11$$

$$14) \frac{4|8x - 16| - 5}{7} = 9$$

Solve each equation:

1) $4|6x - 12| + 9 = 129$

2) $\frac{1}{2} |8x + 4| - 7 = 27$

3) $-7|16x - 8| + 35 = -245$

4) $\frac{2|6x - 9|}{5} + 34 = 4$

$$5) \frac{|4x - 24|}{9} = 8$$

$$6) \frac{3|7x + 28|}{8} - 4 = 17$$

$$7) \frac{3|15 - 5x| + 12}{13} = 9$$

$$8) \frac{5|11x + 33| - 12}{4} = 52$$

9) $-2|\frac{1}{2}x + 8| - 5 = -25$

10) $6|\frac{1}{4}x - 4| - 73 = -13$

11) $4|5x - 10| + 52 = 12$

12) $\frac{3|12x - 36|}{8} - 7 = 11$

$$13) \frac{3|4x + 32| + 4}{10} = 16$$

$$14) \frac{8|9x - 9| + 3}{15} = 5$$

$$15) \frac{2|35 - 7x|}{14} - 26 = -16$$

$$16) \frac{6|2x - 14| - 12}{4} + 11 = 53$$

17) $-2|\frac{1}{3}x - 12| - 5 = -53$

18) $\frac{2}{3}|12 - \frac{1}{2}x| - 13 = 5$

19) $4|3x + 18| + 140 = 8$

20) $\frac{|16x - 4|}{9} - 7 = -3$

Answer Key:

- | | | | |
|------------------------|-----------------------|----------------------|-----------------------|
| 1) $x = \{-3, 7\}$ | 2) $x = \{-9, 8\}$ | 3) $x = \{-2, 3\}$ | 4) No Solution |
| 5) $x = \{-12, 24\}$ | 6) $x = \{-12, 4\}$ | 7) $x = \{-4, 10\}$ | 8) $x = \{-7, 1\}$ |
| 9) $x = \{-36, 4\}$ | 10) $x = \{-24, 56\}$ | 11) No Solution | 12) $x = \{-1, 7\}$ |
| 13) $x = \{-21, 5\}$ | 14) $x = \{0, 2\}$ | 15) $x = \{-5, 15\}$ | 16) $x = \{-8, 22\}$ |
| 17) $x = \{-36, 108\}$ | 18) $x = \{-30, 78\}$ | 19) No Solution | 20) $x = \{-2, 2.5\}$ |

Q3 Quiz 6 Review

Do all problems in NB:

1) $4|3x - 12| + 5 = 65$

2) $6|10x + 25| - 7 = 143$

3) $2|16x - 48| + 38 = 6$

4) $\frac{2|15x - 30|}{5} - 34 = -4$

$$5) \frac{|8x + 2|}{-11} = -6$$

$$6) \frac{4|8x + 12|}{5} - 18 = -2$$

$$7) \frac{3|7x - 35| - 1}{2} = 10$$

$$8) \frac{5|8x - 4| - 8}{4} = 63$$

9) $-2|6x + 18| - 5 = -29$

10) $\frac{2}{3}|6x + 12| - 21 = -5$

11) $4|5x - 10| + 174 = 14$

12) $\frac{3|11x + 33|}{8} - 7 = 26$

$$13) \frac{3|10 - 2x| + 2}{5} = 16$$

$$14) \frac{5|15 - 5x| + 8}{4} = 52$$

$$15) \frac{2|6x - 24|}{-9} + 5 = 17$$

$$16) \frac{6|6x - 6| + 4}{5} + 12 = 56$$

Answer Key:

- | | | | |
|-----------------------|----------------------|------------------------------|----------------------|
| 1) $x = \{-1, 9\}$ | 2) $x = \{-5, 0\}$ | 3) No Solution | 4) $x = \{-3, 7\}$ |
| 5) $x = \{-8.5, 8\}$ | 6) $x = \{-4, 1\}$ | 7) $x = \{4, 6\}$ | 8) $x = \{-6, 7\}$ |
| 9) $x = \{-5, -1\}$ | 10) $x = \{-6, 2\}$ | 11) No Solution | 12) $x = \{-11, 5\}$ |
| 13) $x = \{-18, -8\}$ | 14) $x = \{-5, 11\}$ | 15) $x = \text{No Solution}$ | 16) $x = \{-5, 7\}$ |

Radical Equations

If you have the variable (usually x) under the radical, we cannot solve for it until we get it out from under the radical. The way to do this is to get the expression with the radical by itself and then square both sides. When the radical is by itself, squaring it gets rid of the radical. At this level, we will wind up with either a linear equation or a quadratic equation (both of which we covered in this packet).

Example 1: $7\sqrt{11x+3} - 46 = -4$

$$\begin{array}{r} 7\sqrt{11x+3} - 46 = -4 \\ \quad \quad \quad +46 \quad \quad +46 \end{array}$$

$$\frac{7\sqrt{11x+3}}{7} = \frac{42}{7}$$

$$\begin{array}{l} \sqrt{11x+3} = 6 \quad \text{*Radical is isolated, square both sides.. this gets rid of the radical on the left side of the equation.} \\ (\sqrt{11x+3})^2 = (6)^2 \end{array}$$

$$\begin{array}{r} 11x+3 = 36 \\ \quad \quad -3 \quad \quad -3 \end{array}$$

$$\frac{11x}{11} = \frac{33}{11}$$

$$x = 3 \quad \text{*You MUST do your check to make sure your solution is not an extraneous root.}$$

Check: $7\sqrt{11x+3} - 46 = -4 \quad x = 3$

$$\begin{array}{r} 7\sqrt{11(3)+3} - 46 = -4 \\ 7\sqrt{33+3} - 46 = -4 \\ 7\sqrt{36} - 46 = -4 \\ 7(6) - 46 = -4 \\ 42 - 46 = -4 \\ -4 = -4 \quad \checkmark \end{array}$$

$$x = \{3\}$$

Example 2:

$$\sqrt{13x + 30} - x = 0$$

$$\sqrt{13x + 30} \stackrel{+x}{=} x \stackrel{+x}{}$$

$$(\sqrt{13x + 30})^2 = (x)^2 \quad (\text{Radical is isolated, square both sides})$$

$$\begin{array}{r} 13x + 30 = x^2 \\ \hline -13x - 30 \quad -13x - 30 \end{array}$$

$$0 = x^2 - 13x - 30 \quad (\text{Factor and solve})$$

$$0 = (x + 2)(x - 15)$$

$$x + 2 = 0 \quad x - 15 = 0$$

$$\begin{array}{r} -2 \quad -2 \quad +15 \quad +15 \\ \hline x = -2 \quad x = 15 \end{array} \quad (\text{Check for extraneous roots})$$

Check:

$$\sqrt{13x + 30} - x = 0$$

$$x = -2$$

$$\begin{array}{r} \sqrt{13(-2) + 30} - (-2) = 0 \\ \sqrt{-26 + 30} - (-2) = 0 \\ \sqrt{4} - (-2) = 0 \\ 2 - (-2) = 0 \\ 4 \neq 0 \end{array}$$

$$x = 15$$

$$\begin{array}{r} \sqrt{13(15) + 30} - (15) = 0 \\ \sqrt{195 + 30} - (15) = 0 \\ \sqrt{225} - (15) = 0 \\ 15 - (15) = 0 \\ 0 = 0 \quad \checkmark \end{array}$$

$$x = \{15\}$$

Example 3: $\sqrt{12x - 20} - x = 1$
 $\quad\quad\quad\quad\quad +x \quad +x$

$\sqrt{12x - 20} = x + 1$ *(Radical is isolated, square both sides)*

$$\begin{array}{rcl} (\sqrt{12x - 20})^2 & = & (x + 1)^2 \\ (\sqrt{12x - 20})^2 & = & (x + 1)(x + 1) \\ 12x - 20 & = & x^2 + 1x + 1x + 1 \\ 12x - 20 & = & x^2 + 2x + 1 \\ \hline -12x + 20 & & -12x + 20 \end{array}$$

$0 = x^2 - 10x + 21$ *(Factor and solve)*

$0 = (x - 3)(x - 7)$
 $x - 3 = 0 \quad x - 7 = 0$
 $\quad +3 \quad +3 \quad +7 \quad +7$
 $x = 3 \quad x = 7$ *(Check for extraneous roots)*

Check:

$\sqrt{12x - 20} - x = 1$

$x = 3$

$x = 7$

$$\begin{array}{rcl} \sqrt{12(3) - 20} - (3) & = & 1 \\ \sqrt{36 - 20} - (3) & = & 1 \\ \sqrt{16} - (3) & = & 1 \\ 4 - (3) & = & 1 \\ 1 & = & 1 \quad \checkmark \end{array}$$

$$\begin{array}{rcl} \sqrt{12(7) - 20} - (7) & = & 1 \\ \sqrt{84 - 20} - (7) & = & 1 \\ \sqrt{64} - (7) & = & 1 \\ 8 - (7) & = & 1 \\ 1 & = & 1 \quad \checkmark \end{array}$$

$x = \{3, 7\}$

Example 4: $\sqrt{11x + 36} + 6 = x$
 $\quad\quad\quad\quad -6 \quad\quad\quad -6$

$\sqrt{11x + 36} = x - 6$ (*Radical is isolated, square both sides*)

$(\sqrt{11x + 36})^2 = (x - 6)^2$
 $(\sqrt{11x + 36})^2 = (x - 6)(x - 6)$
 $11x + 36 = x^2 - 6x - 6x + 36$
 $11x + 36 = x^2 - 12x + 36$
 $\underline{-11x - 36} \quad\quad\quad \underline{-11x - 36}$

$0 = x^2 - 23x$ (*Factor and solve*)

$0 = x(x - 23)$
 $x = 0 \quad x - 23 = 0$
 $\quad\quad\quad \underline{+23} \quad \underline{+23}$
 $\quad\quad\quad x = 23$

Check:

$\sqrt{11x + 36} + 6 = x$

$x = 0$

$x = 7$

$\sqrt{11(0) + 36} + 6 = (0)$
 $\sqrt{0 + 36} + 6 = 0$
 $\sqrt{36} + 6 = 0$
 $6 + 6 = 0$
 $12 \neq 0 \quad \checkmark$

$\sqrt{11(23) + 36} + 6 = (23)$
 $\sqrt{253 + 36} + 6 = 23$
 $\sqrt{289} + 6 = 23$
 $17 + 6 = 23$
 $23 = 23 \quad \checkmark$

$x = \{23\}$

1) $\sqrt{14x + 1} = 13$

$$2) 3\sqrt{8x + 9} - 24 = -3$$

$$3) \sqrt[3]{21 - 4x} + 17 = 20$$

$$4) \sqrt{72 - x} = x$$

$$5) \sqrt{-19x - 60} = x$$

$$6) -\sqrt{-19x - 60} = x$$

$$7) \sqrt{16x - 48} = x$$

$$8) \sqrt{12x + 4} - 3 = x$$

$$9) \sqrt{11x + 26} - x = 4$$

10) $\sqrt{12x + 4} - 2 = x$

11) $\sqrt{8x + 1} - x = -1$

12) $\sqrt{41 - 2x} - 11 = x$

13) $\sqrt{8x + 25} - x = -5$

$$14) \sqrt{-28 - 11x} = x$$

$$15) \sqrt{13x + 61} - 7 = x$$

Q3 Quiz 7 Review:

$$1) \sqrt{7x - 6} = x$$

$$x = \{1, 6\}$$

$$2) 6\sqrt{6x - 2} - 55 = -7$$

$$x = 11$$

$$3) \frac{1}{2} \sqrt{11x + 31} + 9 = 16$$

$$x = 15$$

$$4) \sqrt{x^2 - 48} + 2 = x$$

$$x = 13$$

5) $\sqrt{5x + 4} - x = -2$

$x = \{9\}$, 0 is an extraneous root

6) $\sqrt{14x - 6} - x = 3$

$x = \{3, 5\}$

7) $\sqrt{10x - 1} + 4 = x$

$x = \{17\}$, 1 is an extraneous root

8) $\sqrt{9x + 1} - x = -1$

$x = \{11\}$, 0 is an extraneous root

$$9) \sqrt[3]{4} \sqrt{13x - 38} - 8 = 1$$

$$x = 14$$

$$10) \sqrt{11x + 49} - 7 = x$$

$$x = \{-3, 0\}$$

