Chapter 9:
Roots and Irrational Numbers

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Algebra Square Roots 9A

Square roots, cube roots, and higher level roots are important mathematical tools because they are the **inverse operations** to the operations of **squaring and cubing**. In this unit we will study these operations, as well as numbers that come from using them. First, some basic review of what you've seen before.

Exercise #1: Find the value of each of the following **principal square roots**. Write a reason for your answer in terms of a multiplication equation.

(a) $\sqrt{25}$ (b) $\sqrt{9}$ (c) $\sqrt{100}$ (d) $\sqrt{0}$ (e) $\sqrt{\frac{1}{4}}$ (f) $\sqrt{\frac{64}{9}}$

It is generally agreed upon that all **positive, real numbers** have two square roots, a positive one and a negative one. We simply designate which one we want by either including a negative sign or leaving it off.

Exercise #2: Give all square roots of each of the following numbers.

(a) 4 (b) 36 (c) $\frac{1}{16}$

Exercise #3: Given the function $f(x) = \sqrt{x+3}$, which of the following is the value of $f(46)$?

(1) 22 (3) 16

(2) 5 (4) 7

Square roots have an interesting property when it comes to multiplication. Lets discover that property.

Exercise #4: Find the value of each of the following products.

(a) $\sqrt{4} \cdot \sqrt{9}$ (b) $\sqrt{4 \cdot 9}$ (c) $\sqrt{4} \cdot \sqrt{25}$ (d) $\sqrt{4 \cdot 25}$

What you should notice in the last exercise is the following important property of square roots.

MULTIPLICATON PROPERTY OF SQUARE ROOTS

1. $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ LIKEWISE **2.** $\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$

One obvious use for this is to multiply two "***unfriendly***" square roots to get a nice result.

***Exercise* #5:** Find the result of each of the following products.

(a) $\sqrt{2} \cdot \sqrt{8}$ (b) $\sqrt{12} \cdot \sqrt{3}$ (c) $\sqrt{20} \cdot \sqrt{5}$

One less obvious use for the square root property above is in **simplifying square roots of non-perfect squares**. This is a fairly antiquated skill that is almost completely irrelevant to algebra, but it often arises on standardized tests and thus is a good skill to become fluent with.

***Exercise* #6:** To introduce **simplifying square roots**, let's do the following first.

(a) List out the first 10 perfect squares (starting with 1).

Perfect Squares:	Square Root Simplified:
1	
4	
9	

(b) Now consider $\sqrt{18}$. Which of these perfect squares is a factor of 18?

(c) Simplify the $\sqrt{18}$. This is known as writing the answer in simplest radical form.

The key to simplifying any square root is to find the **largest perfect square** that is a factor of the **radicand** (the number under the square root).

Example #1 – Simplify $\sqrt{48}$

$\sqrt{48}$ 1. Put the number in y = screen and divide by x. Then use the table to find the factors. Using only the Y – column, find the largest perfect square which divides evenly into the given number

$\sqrt{16}\,\sqrt{3}$ 2. Write the number appearing under your radical as the product of the perfect square and your answer from the division. Give each number in the product its own radical sign.

$4\sqrt{3}$ 3. Reduce the "perfect" radical that you created. Now you have simplified your radical.

Example #2 – Simplify $3\sqrt{50}$

$3\sqrt{50}$ 1. Don't let the number in front of the radical distract you. It is simply "along for the ride" and will be multiplied times our final answer.

$3\sqrt{25}\sqrt{2}$ 2. Reduce the "perfect" radical

$3 \cdot 5\sqrt{2}$ 3. Multiply the reduced radical by the 3 (who is "along for the ride")

$15\sqrt{2}$ 4. Box your final answer.

Exercise #7: Write each of the following square roots in simplest radical form.

(a) $\sqrt{8}$

(b) $\sqrt{45}$

(c) $\sqrt{48}$

(d) $-2\sqrt{75}$

(e) $\sqrt{72}$

(f) $-\sqrt{500}$

(g) $\sqrt{320}$

(h) $\sqrt{250}$

(i) $\sqrt{200}$

(j) $6\sqrt{750}$

(k) $-11\sqrt{54}$

(l) $4\sqrt{180}$

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Algebra Square Roots 9A HW

1. Simplify each of the following. Each will result in a rational number answer.

(a) $\sqrt{36}$ (b) $-\sqrt{4}$ (c) $\sqrt{121}$ (d) $\sqrt{\frac{1}{9}}$

(e) $-\sqrt{100}$ (f) $\sqrt{\frac{81}{36}}$ (g) $-\sqrt{\frac{1}{16}}$ (h) $-\sqrt{144}$

2. Find the final, simplified answer to each of the following by evaluating the square roots first. Show the steps that lead to your final answers.

(a) $\sqrt{9}+\sqrt{25}-\sqrt{64}$ (b) $5\sqrt{4}+2\sqrt{81}$

(c) $\frac{2\sqrt{25}+2}{3}$ (d) $\sqrt{\frac{1}{4}}(\sqrt{121}-\sqrt{9})$

All of the square roots so far have been "nice." We will discuss what this means more in the next lesson. We can use the Multiplication Property to help simplify certain products of not-so-nice square roots.

3. Find each of the following products by first multiplying the **radicands** (the numbers under the square roots).

(a) $\sqrt{2}\cdot\sqrt{50}$ (b) $\sqrt{3}\cdot\sqrt{12}$ (c) $5\sqrt{6}\cdot\sqrt{24}$

(d) $\sqrt{25}-\sqrt{2}\cdot\sqrt{8}$ (e) $\sqrt{\frac{1}{2}}\cdot\sqrt{\frac{1}{18}}$ (f) $\sqrt{\frac{3}{4}}\cdot\sqrt{\frac{27}{4}}$

4. Write each of the following in **simplest radical form**. Show the work that leads to your answer. The first exercise has been done to remind you of the procedure.

(a) $\sqrt{162}$

$\sqrt{81}\ \sqrt{2}$

$\boxed{9\sqrt{2}}$

(b) $\sqrt{20}$

(c) $-\sqrt{90}$

(d) $\sqrt{48}$

(e) $-\sqrt{8}$

(f) $\sqrt{300}$

5.Write each of the following products in **simplest radical form**. The first is done as an example for you.

(a) $3\sqrt{12}$

$3\ \sqrt{4}\ \sqrt{3}$

$3\cdot 2\ \sqrt{3}$

$\boxed{6\sqrt{3}}$

(b) $4\sqrt{45}$

(c) $\frac{1}{2}\sqrt{32}$

(d) $-2\sqrt{288}$

(e) $\frac{\sqrt{108}}{3}$

(f) $\frac{-\sqrt{320}}{16}$

Reasoning: It is critical to understand that when we "simplify" a square root or perform any calculation using them, we are always finding **equivalent numerical expressions**. Let's make sure we see that in the final exercise.

6. Consider $\sqrt{28}$.

(a) Use your calculator to determine its value. Round your answer to the nearest hundredth.

(b) Write $\sqrt{28}$ in simplest radical form.

(c) Use your calculator to find the value of the product from part (b). How does it compare to your answer from part (a)?

Review Section:

____ 7.) If $A = 3x^2 + 5x - 6$ and $B = -2x^2 - 6x + 7$, then $A - B$ equals

(1) $-5x^2 - 11x + 13$ (3) $-5x^2 - x + 1$

(2) $5x^2 + 11x - 13$ (4) $5x^2 - x + 1$

____ 8.) The table below shows the average yearly balance in a savings account where interest is compounded annually. No money is deposited or withdrawn after the initial amount is deposited.

Year	Balance, in Dollars
0	380.00
10	562.49
20	832.63
30	1232.49
40	1824.39
50	2700.54

Which type of function best models the given data?

(1) linear function with a negative rate of change

(2) linear function with a positive rate of change

(3) exponential decay function

(4) exponential growth function

9.) The breakdown of a sample of a chemical compound is represented by the function $p(t) = 300(0.5)^t$, where $p(t)$ represents the number of milligrams of the substance and t represents the time, in years. In the function $p(t)$, explain what 0.5 and 300 represent.

10.) Factor the expression $x^4 + 6x^2 - 7$ completely.

Homework Answers

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Algebra Square Roots 9A HW

1.) a.) 6 b.) -2 c.) 11 d.) $\frac{1}{3}$

e.) -10 f.) $\frac{3}{2}$ g.) $-\frac{1}{4}$ h.) -12

2.) a.) 0 b.) 28

c.) 4 d.) 4

3.) a.) 10 b.) 6 c.) 60

d.) 1 e.) $\frac{1}{6}$ f.) $\frac{9}{4}$

4.) b.) $2\sqrt{5}$ c.) $-3\sqrt{10}$

d.) $4\sqrt{3}$ e.) $-2\sqrt{2}$ f.) $10\sqrt{3}$

5.) b.) $12\sqrt{5}$ c.) $2\sqrt{2}$

d.) $-24\sqrt{2}$ e.) $2\sqrt{3}$ f.) $-\frac{1}{2}\sqrt{5}$

6.) a.) 5.29 b.) $2\sqrt{7}$ c.) equal

7.) (2)

8.) (4)

9.) 50% decay and 300 is the initial amount

10.) $(x^2+7)(x+1)(x-1)$

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Algebra Irrational Numbers 9B

The set of real numbers is made up of two distinctly different numbers. Those that are **rational** and those that are **irrational**. Their technical definitions are given below.

RATIONAL AND IRRATIONAL NUMBERS

1. A **rational rumber** is any number that can be written as the **ratio** of **two integers**. Such numbers include $\frac{3}{4}$, $\frac{-7}{3}$, and $\frac{5}{1}$. These numbers have **terminating** or **repeating decimals**.
2. An **irrational number** is any number that is **not rational**. So, ones that cannot be written as the ratio of two integers. These numbers have **nonterminating and nonrepeating decimal representations.**

Exercise #1: Let's consider a number that is rational and one that is irrational (**not rational**). Consider the rational number $\frac{2}{3}$ and the irrational number $\sqrt{\frac{1}{2}}$. Both of these numbers are less than 1.

(a) Draw a pictorial representation of $\frac{2}{3}$ of the rectangle shown below.

(b) Using your calculator, give the decimal representation of the number $\frac{2}{3}$. Notice that it has a repeating pattern.

(c) Write out all the decimal places that your calculator gives you for the $\sqrt{\frac{1}{2}}$. Notice that it does not have a repeating pattern.

(d) Why could you not draw a pictorial representation for $\sqrt{\frac{1}{2}}$ the same way that you do for $\frac{2}{3}$?

Irrational numbers are necessary for a variety of reasons, but they are somewhat of a mystery. In essence they are a number that can never be found by **subdividing** an **integer quantity** into a **whole number** of **parts** and then taking an **integer number** of those parts. There are many, many types of irrational numbers, but **square roots of non-perfect squares** are **always irrational**. The proof of this is beyond the scope of this course.

Exercise #2: Write out every decimal your calculator gives you for these **irrational numbers** and notice that they never repeat.

(a) $\sqrt{2}$ = ____

(b) $\sqrt{10}$ = ____

(c) $\sqrt{23}$ = ____

Rational and irrational numbers often mix, as when we simplify the square root of a non-perfect square.

Exercise #3: Consider the **irrational** number $\sqrt{28}$.

(a) Without using your calculator, between what two consecutive integers will this number lie? Why?

(b) Using your calculator, write out all the decimals for $\sqrt{28}$.

(c) Write $\sqrt{28}$ in simplest radical form.

(d) Write out the decimal representation for your answer from part (c). Notice it is the same as (b).

So, it appears that a **non-zero rational number times an irrational number** (see letter (c) above) results in an **irrational number** (see letter (d) above). We should also investigate what happens when we add rational numbers to irrational numbers (and subtract them).

Exercise #4: For each of the following addition or subtraction problems, a rational number has been added to an irrational number. Write out the decimal representation that your calculator gives you and classify the result as rational (if it has a repeating decimal) or irrational (if it doesn't).

(a) $\frac{1}{2} + \sqrt{2}$

(b) $\frac{4}{3} + \sqrt{10}$

(c) $7 - \sqrt{8}$

Exercise #5: Fill in the following statement about the sum or rational and irrational numbers.

> When a **rational number** is added to an **irrational number** the result is always ______________________.

Exercise #6: Which of the following is an irrational number? If necessary, play around with your calculator to see if the decimal representation does not repeat. **Don't be fooled by the square roots**.

(1) $\sqrt{25}$

(3) $\frac{7}{2}$

(2) $4 - \sqrt{9}$

(4) $3 + \sqrt{6}$

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Algebra Irrational Numbers 9B HW

1. For each of the following rational numbers, use your calculator to write out either the terminating decimal or the repeating decimal patterns.

(a) $\frac{3}{4}$ (b) $\frac{4}{9}$ (c) $\frac{5}{8}$ (d) $\frac{5}{6}$

(e) $\sqrt{\frac{25}{4}}$ (f) $\sqrt{\frac{1}{100}}$ (g) $\sqrt{\frac{4}{9}}$ (h) $\sqrt{\frac{2}{32}}$

2. One of the most famous **irrational numbers** is the number pi, π , which is essential in calculating the circumference and area of a circle.

(a) Use your calculator to write out all of the decimals your calculator gives you for π. Notice no repeating pattern.

(b) Historically the rational number $\frac{22}{7}$ has been used to ***approximate*** the value of π. Use your calculator to write out all the decimals for this rational number and compare it to part (a).

3. For each of the following irrational numbers, do two things: (1) write the square root in simplest radical form and then (2) use your calculator to write out the decimal representation.

(a) $\sqrt{32}$ (b) $\sqrt{98}$ (c) $\sqrt{75}$

(d) $\sqrt{500}$ (e) $\sqrt{80}$ (f) $\sqrt{117}$

Reasoning: Types of numbers mix and match in various ways. The last exercise shows us a trend that we explored during the lesson.

4. Fill in the statement below based on the last exercise with one of the words below the blank.

- The product of a (non-zero) rational number and an irrational number results in a(n) ________________ number.
  rational or irrational

5. Let's explore the **product** of **two irrational numbers** to see if it is **always irrational, sometimes irrational, sometimes rational,** or **always rational.** Find each product below using your calculator (be careful as you put it in) and write out all decimals. Then, classify as either rational or irrational.

(a) $\sqrt{5} \cdot \sqrt{3} =$ ______________________________ Rational or irrational?

(b) $\sqrt{8} \cdot \sqrt{18} =$ ______________________________ Rational or irrational?

(c) $\sqrt{7} \cdot \sqrt{11} =$ ______________________________ Rational or irrational?

(d) $\sqrt{11} \cdot \sqrt{11} =$ ______________________________ Rational or irrational?

6. Based on #5, classify the following statement as true or false (you must write the whole word):

- The product of two irrational number is always irrational. ____________________

7. Let's explore adding rational numbers. Using what you learned about in middle school, add each of the following pairs of rational numbers by first finding a **common denominator** then combine. Then, determine their repeating or terminating decimal.

(a) $\frac{1}{2}+\frac{2}{3}$ (b) $\frac{3}{4}+\frac{1}{2}$ (c) $\frac{3}{8}+\frac{5}{12}$

(d) Classify the following statement as true or false (you must write the whole word):

- The sum of two rational numbers is always rational. ____________________

8. Finally, what happens when we add a rational and an irrational number (we explored this in Exercises #4 through #6 in the lesson). Fill in the blank below from what you learned in class.

- The sum of a rational number with an irrational number will always give a(n) ____________________ number.
  rational or irrational

Review Section:

____ 9.) A satellite television company charges a one-time installation fee and a monthly service charge. The total cost is modeled by the function $y = 40 + 90x$. Which statement represents the meaning of each part of the function?

(1) y is the total cost, x is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

(2) y is the total cost, x is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.

(3) x is the total cost, y is the number of months of service, \$40 is the installation fee, and \$90 is the service charge per month.

(4) x is the total cost, y is the number of months of service, \$90 is the installation fee, and \$40 is the service charge per month.

___ 10.) If $4x^2 - 100 = 0$, the roots of the equation are

(1) -25 and 25 (3) -5 and 5

(2) -25, only (4) -5, only

11.) What is the value of x in the equation $\frac{x-2}{3} + \frac{1}{6} = \frac{5}{6}$?

12.) Rhonda deposited \$3000 in an account in the Merrick National Bank, earning 4.2% interest, compounded annually. She made no deposits or withdrawals. Write an equation that can be used to find B, her account balance after t years.

Homework Answers

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Algebra Irrational Numbers 9B HW

1,) a) 0.75 b) $0.\overline{4}$ c) 0.625 d) $0.8\overline{3}$

e) 2.5 f) 0.1 g) $0.\overline{67}$ h) 0.25

2.) a)3.141592654

b) 3.142857143 unequal

3.) a) $4\sqrt{2} = 5.656854249$ b) $7\sqrt{2} = 9.899494937$ c) $5\sqrt{3} = 8.660254038$

d) $10\sqrt{5} = 22.36067977$ e) $4\sqrt{5} = 8.94427191$ f) $3\sqrt{13} = 10.81665383$

4.)Irrational

5.) a) 3.872983346 Irrational

b) 12 Rational

c) 8.774964387 Irrational

d) 11 Rational

6.) False

7.) a) $1.1\overline{6}7$ b) 1.25 c) $0.791\overline{6}7$ d) True

8.) Irrational

9.) (2)

10.) (3)

11.) x = 4

12.) $B = 3000(1 + 0.042)^t$

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Algebra Square Root Functions 9C

Square roots are operations on numbers that give exactly one output for a given input. So, they fit nicely into the definition of a function. We can graph the general square root function, once we establish a **very important fact about square roots**.

Exercise #1: Consider $\sqrt{-4}$?

(a) Why are neither 2 nor -2 the correct square root of -4?

(b) What can you conclude about taking square roots of negative numbers? Explain.

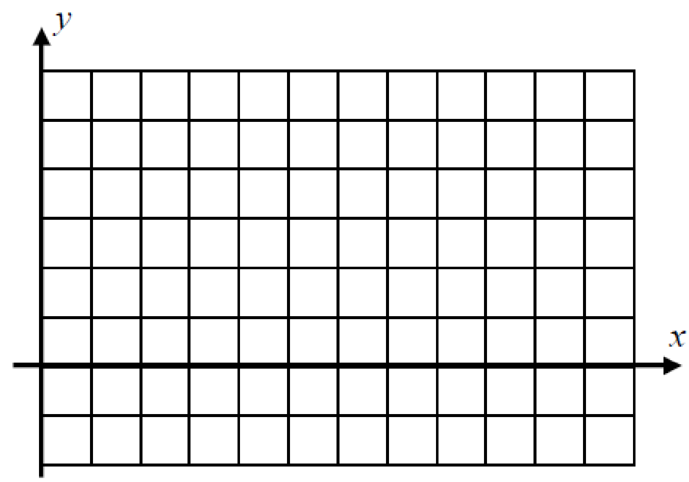
It is absolutely critical that you understand, deep down inside, why **finding the square root of a negative number** is **not possible** with any **real number**. Let's now get into the basic square root graph.

Exercise #2: Consider $f(x) = \sqrt{x}$

(a) Create a table of values for input values of x for which you can find rational square roots.

x				
$f(x)=\sqrt{x}$				

(b) Graph the function on the grid provided.



(c) What is the domain of this function?

(d) What is the range of this function?

(e) Circle the correct choice below that characterizes the function $f(x) = \sqrt{x}$.

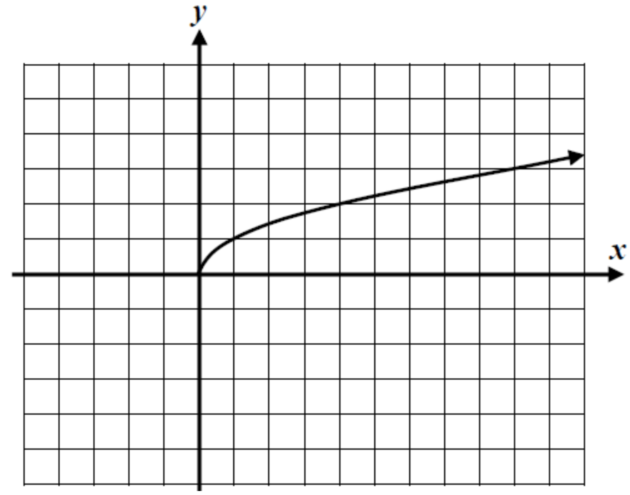
$f(x)$ is always decreasing $f(x)$ is always increasing

(f) What shape does the square root graph appear to be "half" of? **This is not a coincidence.**

Square root graphs can be shifted just as quadratics can. And they shift in much the same way.

Exercise #3: The graph of $y = \sqrt{x}$ is shown below.

(a) Using your calculator, graph the function given by $y = \sqrt{x+4} + 2$. Show your table of values.



x				
y				

(b) State the domain and range of this function.

Domain: ____________________________

Range: ____________________________

(c) Using your calculator, on the same set of axes, graph the function given by $y = \sqrt{x-1} - 4$. Show your table. Also state its domain and range.

Table:

x				
y				

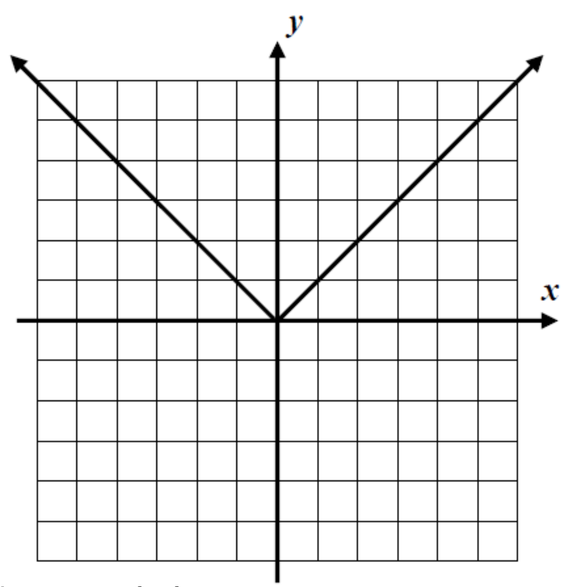
Domain: ____________________________

Range: ____________________________

So, it looks like our the shifting pattern that we saw with quadratics continues to hold with square root functions. This pattern would in fact hold no matter what function we were looking at. For example, let's look back at our friend the absolute value function.

Exercise #4: The graph of $y = |x|$ is shown on the grid below.

(a)Use your calculator to create a graph of $y = |x+3| - 2$. Don't forget your table!



(b) State the domain and range of this function:

Domain: ____________________________

Range: ____________________________

(c) Let's see if you get the pattern. Sketch $y = |x-2| - 1$ without using your calculator.

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Algebra Square Root Functions 9C HW

____ 1. Given the function $f(x) = \sqrt{x-8} + 3$, which of the following is the value of $f(24)$?

(1) 7 (3) 3

(2) 11 (4) 4

____ 2. If $g(x) = 4\sqrt{x}$ then $g(45)$ is

(1) $7\sqrt{5}$ (3) $36\sqrt{5}$

(2) $12\sqrt{5}$ (4) $22\sqrt{5}$

____ 3. Which of the following values of x is *not* in the domain of $y = \sqrt{x-8}$? Remember, the domain is the set of all inputs (x-values) that give an real output (y-value)?

(1) $x = 12$ (3) $x = 8$

(2) $x = 10$ (4) $x = 7$

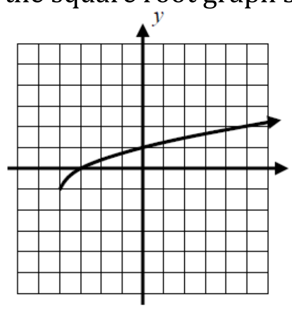
____ 4. Which of the following is the equation of the square root graph shown below?

(1) $y = \sqrt{x+4} + 1$

(2) $y = \sqrt{x+4} - 1$

(3) $y = \sqrt{x-4} - 1$

(4) $y = \sqrt{x-4} + 1$



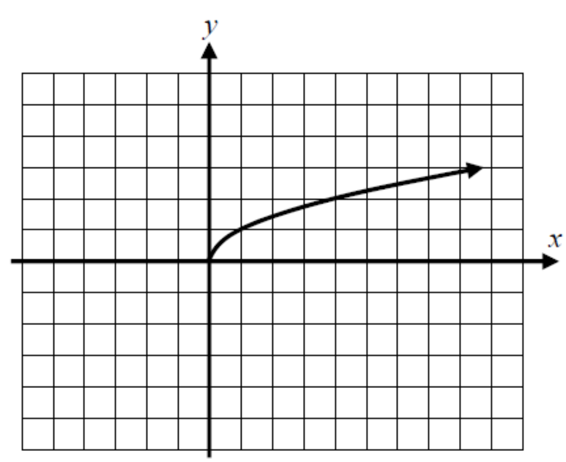
____ 5. Which of the following gives the range of the function $y = |x-1| + 7$? Hint: Create a sketch by hand or on your calculator to help solve this problem.

(1) $y \le 1$ (3) $y \ge 7$

(2) $y \ge 1$ (4) $y \le 7$

6. On the grid shown to the right, $y = \sqrt{x}$ is graphed. Without using your calculator, create a table and graph $y = -\sqrt{x}$ on the same set of axes.

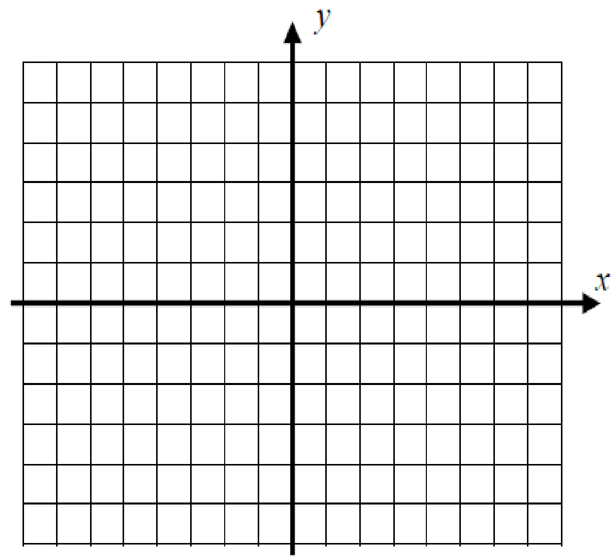
x	0	1	4	9
$y = -\sqrt{x}$				



Explain the effect on the graph of $y = \sqrt{x}$ by multiplying by -1.

7. Graph the function $f(x) = -\sqrt{x+3} + 2$ on the grid below. Show the table that you created by hand or using your calculator. Then, state its domain and range.

Table:



Domain: ______________________________

Range: ______________________________

Review Section:

____ 8.) Which equation has the same solutions as $2x^2 + x - 3 = 0$?

(1) $(2x - 1)(x + 3) = 0$

(2) $(2x + 1)(x - 3) = 0$

(3) $(2x - 3)(x + 1) = 0$

(4) $(2x + 3)(x - 1) = 0$

____ 9.) How does the graph of $f(x) = 3(x - 2)^2 + 1$ compare to the graph of $g(x) = x^2$?

(1) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.

(2) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

(3) The graph of $f(x)$ is narrower than the graph of $g(x)$, and its vertex is moved to the left 2 units and up 1 unit.

(4) The graph of $f(x)$ is wider than the graph of $g(x)$, and its vertex is moved to the right 2 units and up 1 unit.

10.) Solve the equation $4x^2 - 12x = 7$ algebraically for x.

Name: *Homework Answers* Date: __________ Period: _____

Algebra Square Root Functions 9C HW

1.) 1

2.) 2

3.) 4

4.) 2

5.) 3

6.) GRAPH - It was reflected or flipped over the x-axis

7.) GRAPH - Domain: $-3 \leq x < \infty$ - Range: $-\infty < y \leq 2$

8.) 4

9.) 2

10.) $x = \left\{-\frac{1}{2}, \frac{7}{2}\right\}$

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Algebra Finding Zeros by Comp. Squ 9D

In a previous lesson, we saw how to find the **zeroes** of a **quadratic function** if it was in **vertex or shifted form**.

Exercise #1: For the quadratic function $y = 2(x-2)^2 - 36$.

(a) Find the zeros in simplest radical form.

(b) Find the zeros to the nearest tenth.

But, of course, in order for us to find the zeroes using inverse operations as in (a), we need our quadratic in the form $y = a(x-h)^2 + k$. In order to do this, we will use our technique of **Completing the Square**.

Exercise #2: Consider the quadratic $y = x^2 - 6x - 16$.

(a) Find the zeros of this function by factoring.

(b) Find the zeros by Completing the Square.

Now, it would probably seem to many students a bit redundant to know two methods for finding the zeroes of a quadratic function. Let's illustrate why the technique of **Completing the Square** is important in its own right.

Exercise #3: Let's take a look at the quadratic function $y = x^2 + 6x + 2$.

(a) Find the zeros of this function using the method of Completing the square. What kind of numbers are the solutions?

(b) Try to factor $x^2 + 6x + 2$. Show your guesses and checks.

(c) What can you conclude about zeroes that are found using the **Zero Product Law (Factoring)**?

We now have a variety of tools at our disposal to find the **zeroes** and the **turning points** of quadratic functions. In one case we have the factored form of a quadratic; in a second case we have the vertex form of a quadratic. Each has its advantages and disadvantages.

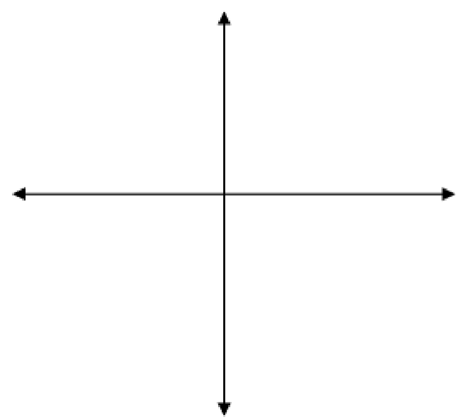
***Exercise* #4:** Let's analyze the quadratic $f(x) = 2x^2 - 4x - 16$, which is written in **standard form**.

(a) Write the function in vertex form and state the coordinates of its turning point.

(b) Using your answer from part (a), find the zeros of the function.

(c) Determine the functions y – intercept.

(d) Draw a rough sketch of the function on the axes below. Label all quantities in part (a) through (c).



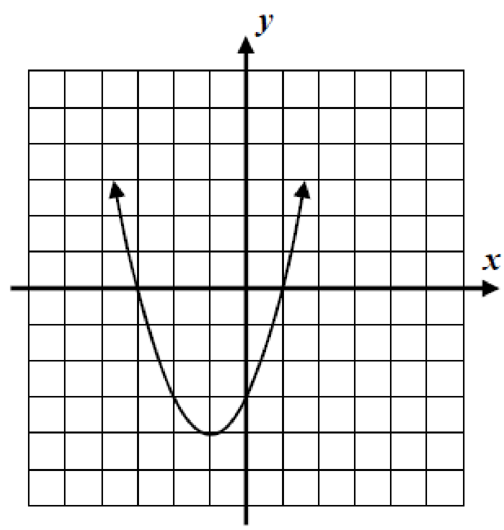
Let's see if we can now go in the opposite direction.

***Exercise* #5:** The quadratic function pictured has a leading coefficient equal to 1. Answer the following questions based on your previous work.

(a) Write the equation of this quadratic in vertex form.

(b) Write the equation of this quadratic in factored form.

(c) How could you establish that these were **equivalent functions**?



Name: ______________________________ Date: ______________ Period: _____

Algebra Finding Zeros by Comp. Squ 9D HW

1. Solve the equation $x^2 - 4x - 12 = 0$ two ways:

(a) By Factoring

(b) By Completing the Square

2. Solve the equation $x^2 + 10x + 21 = 0$ two ways:

(a) By Factoring

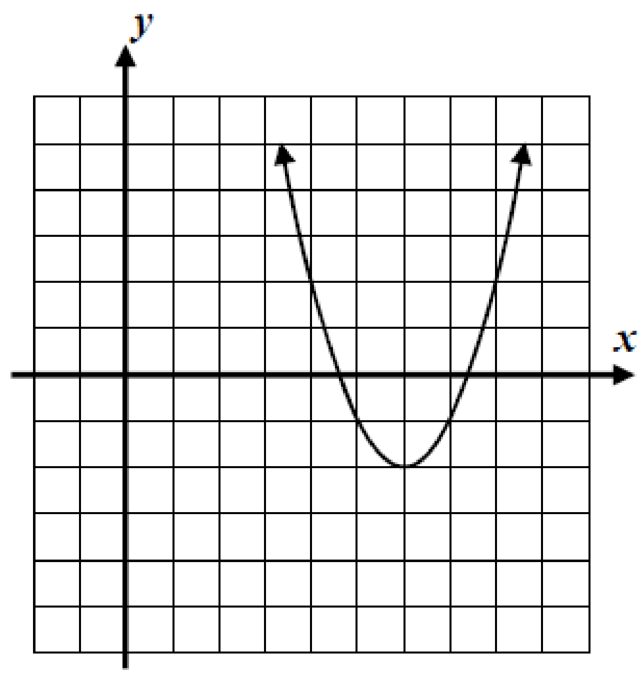
(b) By Completing the Square

3.Find the solutions to the following equation in simplest radical form by using Completing the Square.

$x^2 + 8x - 2 = 0$

4. Using the Method of Completing the square, find the zeroes of the following function to the nearest *hundredth*.
$f(x) = 2x^2 + 12x + 5$

5. Consider the quadratic function shown below whose leading coefficient is equal to 1.
(a) Write the equation of this quadratic in $y = (x - h)^2 + k$ form.



(b) Find the zeroes of this quadratic in simplest radical form.

(c) Write the equation of this quadratic function in
$y = ax^2 + bx + c$, i.e. standard, form.

6. Consider the quadratic function $y = x^2 + 2x - 48$ written in standard form.

(a) Write the quadratic function in its vertex form and state the coordinates of its turning point.

(b) Find the zeros of the function algebraically by setting your equation from part (a) equal to zero.

(c) State the range of this function. Justify your answer by creating a sketch of the function from what you found in part (a) and part (b).

(d) This quadratic can also be written in equivalent factored form as $y = (x-6)(x+8)$. What graphical features are easy to determine when the function is written in this form?

Review Section:

____7.) If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as

(1) $(x - 3y)(x + 3y)$
(2) $(x^2 - 3y)(x^2 + 3y)$
(3) $(x^2 - 3y)(x^2 - 3y)$
(4) $(x^4 + y)(x - 9y)$

____8.) Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for \$1.75 per pound and peaches for \$2.50 per pound. If she made \$337.50, how many pounds of peaches did she sell?

(1) 11
(2) 18
(3) 65
(4) 100

9.) John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x, will they have the same amount of money saved? Explain how you arrived at your answer.

Name: ***Homework Answers*** Date: __________________ Period: ______

Algebra Finding Zeros by Comp. Squ 9D HW

1.) $x = \{-2, 6\}$

2.) $x = \{-7, -3\}$

3.) $x = \{-4 \pm 3\sqrt{2}\}$

4.) $x = \{-5.55, -0.45\}$

5.)a) $y = (x-6)^2 - 2$ $(6, -2)$
b) $x = \{6 \pm \sqrt{2}\}$
c) $y = x^2 - 12x + 34$

6.)a) $y = (x+1)^2 - 49$ $(-1, -49)$
b) $x = \{-8, 6\}$
c) $-49 \leq y < \infty$
d) It is easy to identify the zeros of the function.

7.) 2

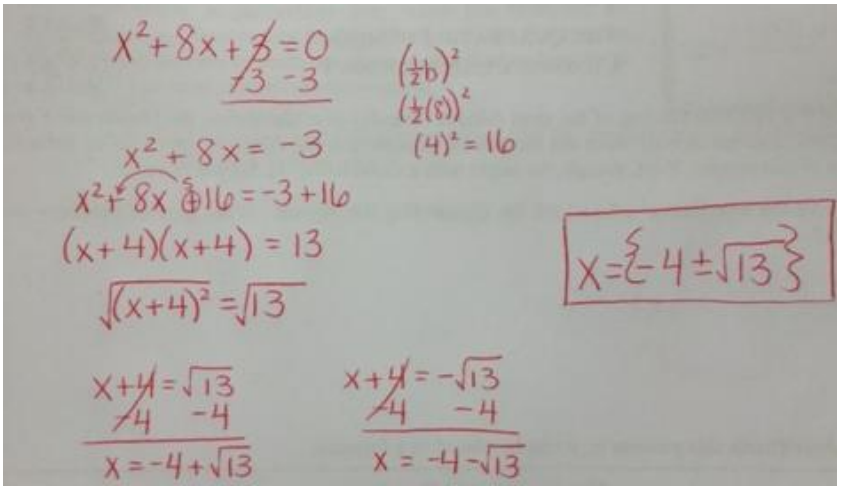
8.) 3

9.) $x = 7$

Name: ____ Date: ________________ Period: _____
Algebra Quadratic Formula 9E

Our final topic in this unit looks at one of the most famous formulas in mathematics, the **Quadratic Formula**. The quadratic formula stems directly from the method of **Completing the Square**. Its proof or derivation is beyond the scope of this course. First, though, we begin with a Completing the Square Problem.

***Exercise* #1:** Solve the equation $x^2+8x+3=0$ by Completing the Square. What type of numbers do your answers represent?



Because these answers take the form of a rational number added to an irrational number, they are irrational zeroes.

Because of how **algorithmic** this process is, it can be placed in a formula:

THE QUADRATIC FORMULA

For the quadratic equation $ax^2+bx+c=0$, the zeroes can be found by $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

***Exercise* #2:** For the previous quadratic $x^2+8x+3=0$ identify the following.

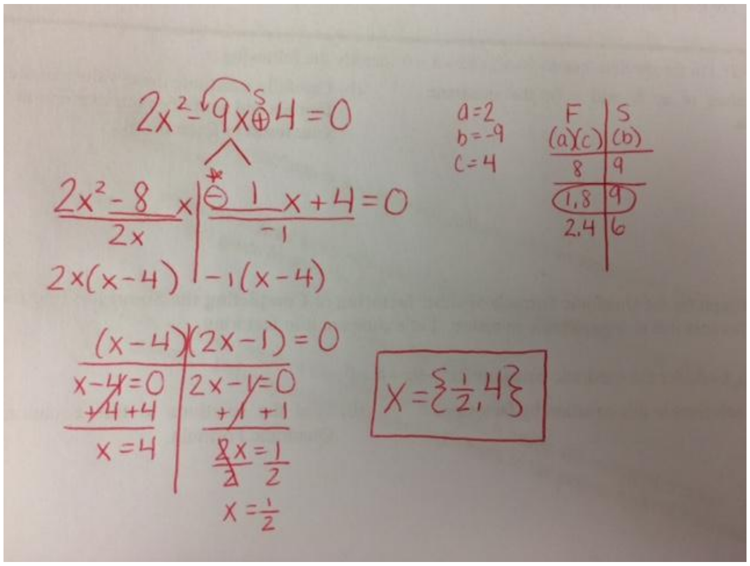
(a) The values of *a*, *b*, and *c* in the quadratic formula.

(b) Carefully substitute these values in the quadratic formula and simplify your expression. Compare your result to Exercise #1.

Students often prefer the Quadratic Formula to either **factoring** or **Completing the Square** to find the zeroes of a quadratic because it is so **algorithmic** in nature. Let's compare it to factoring.

Exercise #3: Consider the quadratic equation $2x^2 - 9x + 4 = 0$.

(a) Find the solutions to this equation by factoring



(b) Find the solutions to this equation using the Quadratic Formula.

The Quadratic Formula is particularly nice when the solutions are **irrational numbers** and thus cannot be found by factoring. Sometimes, we have to place the answers to these equations in **simplest radical form** and sometimes we just need decimal approximations.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Exercise #4: For each of the following quadratic equations, find the solutions using the Quadratic Formula and express your answers in **simplest radical form**.

(a) $x^2 + 6x - 9 = 0$

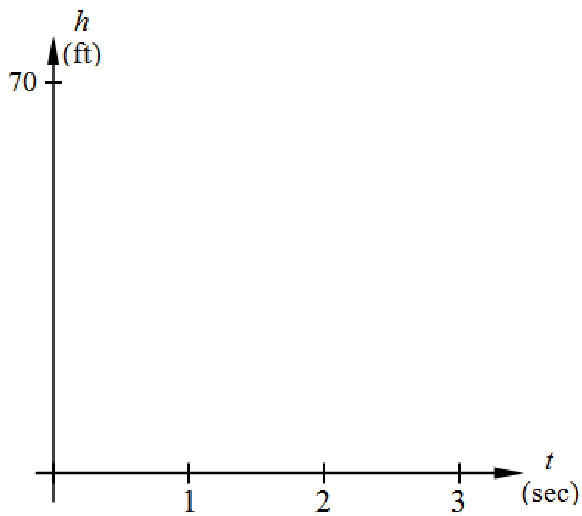
(b) $3x^2 + 4x - 1 = 0$

Many times in applied problems it makes much greater sense to express the answers, even if irrational, as approximated decimals.

***Exercise* #5:** A projectile is fired vertically from the top of a 60 foot tall building. It's height in feet above the ground after t-seconds is given by the formula

$$h = -16t^2 + 20t + 60$$

Using your calculator, sketch a graph of the projectile's height, h, using the indicated window. At what time, t, does the ball hit the ground? Solve by using the quadratic formula to the nearest *tenth* of a second.



Name: ____ Date: _________________ Period: _____
Algebra Quadratic Formula 9E HW

1. Solve the equation $x^2 - 4x - 12 = 0$ by ***factoring only.***

2. Solve the equation $x^2 + 6x + 3 = 0$ by ***completing the square, only***. Express your answer in simplest radical form.

3. Solve the equation $2x^2 - 13x + 20 = 0$ using the ***Quadratic Formula, only***.

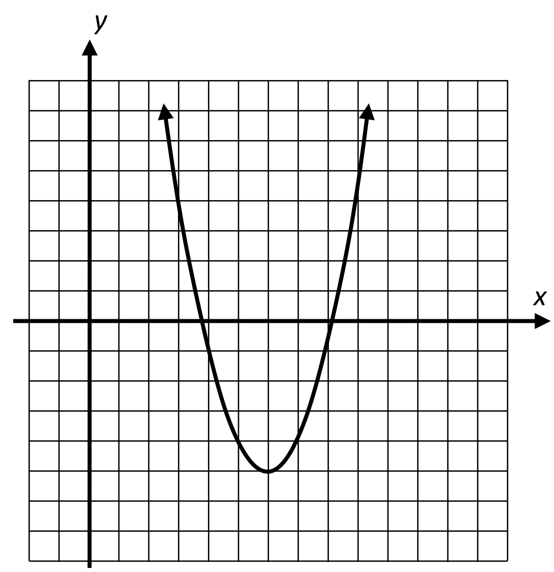
4. Solve the equation $x^2 - 2x = 7$ using the ***Quadratic Formula, only***. Express your answer in simplest radical form.

5. If the quadratic formula is used to solve the equation $x^2 - 4x - 41 = 0$, the correct roots are

(1) $4 \pm 3\sqrt{10}$ (3) $-4 \pm 3\sqrt{10}$

(2) $2 \pm 3\sqrt{5}$ (4) $-2 \pm 3\sqrt{5}$

6. The quadratic function $f(x) = x^2 - 12x + 31$ is shown below.

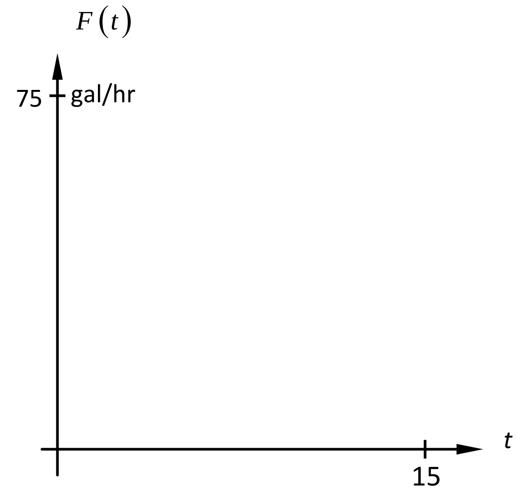


(a) Find the zeroes of this function in simplest radical form by using the quadratic equation.

(b) Write this function in vertex form by completing the square. Based on this, what are the coordinates of its turning point? Verify on the graph.

7. The flow of oil in a pipe, in gallons per hour, can be modeled using the function $F(t) = -2t^2 + 20t + 11$

(a) Using your calculator, sketch the function on the axes provided.



(b) Using the quadratic formula, find, to the nearest tenth of an hour, the time when the flow stops (is zero). Show your work.

(c) Use the process of completing the square to write $F(t)$ in its vertex form. Then, identify the peak flow and the time at which it happens.

Review Section:

8. In the function $f(x) = (x-2)^2 + 4$, the minimum value occurs when x is

(1) -2 (2) 2 (3) -4 (4) 4

There is a question below

9. For a recently released movie, the function $y = 119.67(0.61)^x$ models the revenue earned, y, in millions of dollars each week, x, for several weeks after its release.

Based on the equation, how much more money, in millions of dollars, was earned in revenue for week 3 than for week 5?

(1) 37.27 (2) 27.16 (3) 17.06 (4) 10.11

Name: *Homework Answers* Date: ______________ Period: _____

Algebra Quadratic Formula 9E HW

1.) $x = \{-2,6\}$

2.) $x = -3 \pm \sqrt{6}\}$

3.) $x = \left\{\frac{5}{2}, 4\right\}$

4.) $x = \left\{1 \pm 2\sqrt{2}\right\}$

5.) (2)

6.) (a) $x = \left\{-6 \pm \sqrt{5}\right\}$ (b) $f(x) = (x-6)^2 - 5$ Turning Point: (6,-5)

7.) (a) Sketch (b) 10.5 hours
(c) $F(t) = -2(t-5)^2 + 61$ Vertex: (5,51) Maximum flow: 61 gallons/hour Time of Max: 5 hours

8.) (2)

9.) (3)

Name: ______________________________ Date: ______________ Period: _____

Algebra Quadratic Equations 9F

You now have a large variety of ways to solve quadratic equations, i.e. polynomial equations whose highest powered term is x^2. These techniques include **factoring**, **Completing the Square,** and the **Quadratic Formula**. In each application, it is essential that the equation that we are solving is equal to zero. If it isn't, then some minor manipulation might be needed.

***Exercise* #1:** Solve each of the following quadratic equations using the required method. First, arrange the equations so that they are set equal to zero.

(a) Solve by factoring:

$$x^2+5x-12=8x-2$$

(b) Solve by Completing the Square.

$$x^2-15x+24=-3x+4$$

(c) Solve using the Quadratic Formula
Express answers to the nearest tenth.

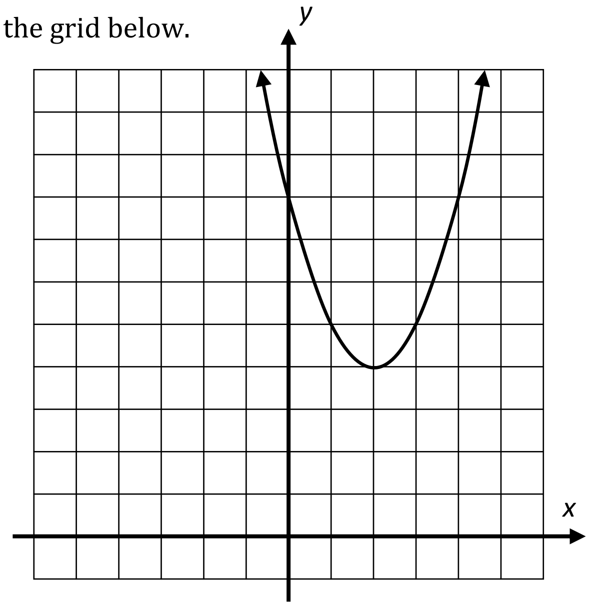
$$x^2-3x+16=5x+15$$

(d) Solve using the Quadratic Formula
Express answers in simplest radical form.

$$x^2+4x+2=-2x+7$$

Our final look at quadratic equations comes as a tie between their zeroes (where the functions cross the x-axis) and the algebraic solutions to find them.

***Exercise* #2:** The quadratic $f(x)=(x-h)^2+k$ is shown graphed on the grid below.



(a) What are the values of h and k?

(b) What happens when you try to solve for the zeroes of f given the values of h and k from part (a)? Why can't you find solutions?

(c) How does what you found in part (b) show up in the graph to the right?

If you think about the graphs of parabolas, they can certainly "miss" the x-axis. When this happens **graphically** then when we solve for the **zeroes algebraically** we won't be able to find any **real solutions** (although perhaps we will find some **imaginary ones** in Algebra II).

***Exercise* #3:** Which of the following three quadratic functions has no real zeroes (there may be more than one). Determine by using the Quadratic Formula. Verify each answer by graphing in the standard viewing window.

$y=x^2+7x+1$ $\qquad$ $y=3x^2+2x+4$ $\qquad$ $y=5x^2+2x-3$

Name: ______________________________ Date: ______________ Period: _____

Algebra Quadratic Equations 9F HW

1. Solve each of the following equations using the method described. Place your final answers in the from asked for.

(a) Solve by factoring:
(Answers are exact)

$$2x^2 - 2x + 1 = 4x + 1$$

(b) Solve by factoring:
(Answers are exact)

$$2x^2 + 5x + 3 = x^2 + 9x + 15$$

(c) Solve by Completing the Square
(Round answers to the nearest *tenth*)

$$x^2 + 10x + 2 = 2x + 5$$

(d) Solve using the Quadratic Formula
(Express answers in simplest radical form)

$$2x^2 + 3x - 3 = -3x - 4$$

2. Which of the following represents the zeroes of the function $f\left(x\right)=x^2-4x+2$?

(1) $\{-1, 2\}$

(2) $\{2-2\sqrt{2}, 2+2\sqrt{2}\}$

(3) $\{2-\sqrt{2}, 2+\sqrt{2}\}$

(4) $\{-1, 4\}$

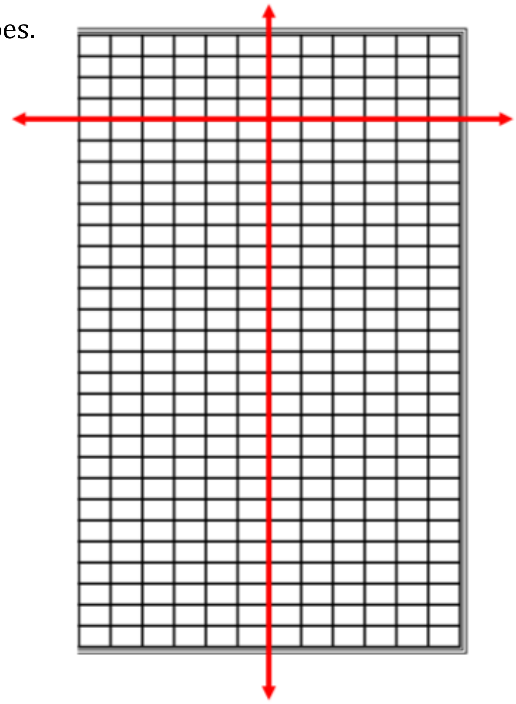
3. The percent of popcorn kernels that will pop, *P*, is modeled using the equation:

$P=-0.03T^2+25T-3600$, where *T* is the temperature in degrees Fahrenheit.

Determine the two temperatures, to the nearest degree Fahrenheit, that result in zero percent of the kernels popping. Use the Quadratic Formula. Show work that justifies your answer. The numbers here will be messy. Use your calculator to help you and carefully write out your work.

4. (a) Find the zeroes of the function $y=x^2-4x-16$ by Completing the square. Express your answers in simplest radical form.

(b) Graph the parabola using a standard window to see the irrational zeroes.
Must include a table with your answer



5. Explain how you can tell that the quadratic function $y = x^2 + 6x + 15$ has no real zeroes *without* graphing the function.

6. Use the Quadratic Formula to determine which of the two functions below would have real zeroes and which would not, then verify by graphing on your calculator using the STANDARD VIEWING WINDOW.

$y = 2x^2 + 3x - 1$ $\qquad\qquad$ $y = x^2 + 2x + 3$

There are questions below

Review Section:

7. Michael borrows money from his uncle, who is charging him simple interest using the formula $I = Prt$. To figure out what the interest rate, r, is, Michael rearranges the formula to find r. What would be his new formula?

8. The equation $A = 1300(1.02)^x$ is being used to calculate the amount of money in a savings account where x represents the number of years. Answer the following questions below:

(a) What is the initial amount of money in the account?

(b) Is the money in the account increasing or decreasing?

(c) Using your answer from part (b), by what percent?

(d) Calculate the amount of money to the nearest cent, after 11 years.

Name: ***Homework Answers*** ______________________ Date: ________________ Period: _____

Algebra Quadratic Equations 9F HW

1.) (a) $x = \{0,3\}$ $(b) x = \{-2,6\}$ (c) $x = \{-8.4, .4\}$ (d) $x = \left\{\frac{-3 \pm \sqrt{7}}{2}\right\}$

2.) (3)

3.) $185°\text{F}$ *and* $648°\text{F}$

4.) (a) $x = \{2 \pm 2\sqrt{5}\}$ (b) Graph with Table

5.) Show that it has no real zeroes with an explanation.

6.) First equation has real zeroes with appropriate work.

7.) $r = \left(\frac{I}{Pt}\right)$

8.) (a) $1,300 (b) Increasing (c) 2% Increase (d) $A = \$1,616.39$

Name: ______________________________________ Date: _______________ Period: _____
Algebra Cube Roots 9G

Just like square roots undo the squaring process, **cube roots**, undo the **process of cubing a number**. The cube root's technical definition along with its symbolism is given below.

CUBE ROOTS

If $x^3 = a$ then $\sqrt[3]{a}$ is a solution to this equation. Or... $\sqrt[3]{a}$ is any number that when cubed gives a.

***Exercise* #1:** It is good to know some basic cube roots of smaller numbers. Find each of the following and justify by using a multiplication statement.

(a) $\sqrt[3]{8}$ (b) $\sqrt[3]{1}$ (c) $\sqrt[3]{27}$

(d) $\sqrt[3]{0}$ (e) $\sqrt[3]{-1}$ (f) $\sqrt[3]{-8}$

One of the most striking differences between **square roots** and **cube roots** is that you can find the **cube root of negative real numbers**. For square roots, that will have to wait until you learn about non-real numbers in Algebra II.

***Exercise* #2:** Using your calculator, use a guess and check scheme to find the following cube roots. Justify using a multiplication statement.

(a) $\sqrt[3]{343}$ (b) $\sqrt[3]{-2744}$ (c) $\sqrt[3]{12{,}167}$

Most calculators have a cube root option, although it may be harder to find than the square root button.

***Exercise* #3:** Find each of the following cube roots to the nearest *tenth* by using your calculator's cube root option/button.

(a) $\sqrt[3]{100}$ (b) $\sqrt[3]{-364}$ (c) $\sqrt[3]{982}$

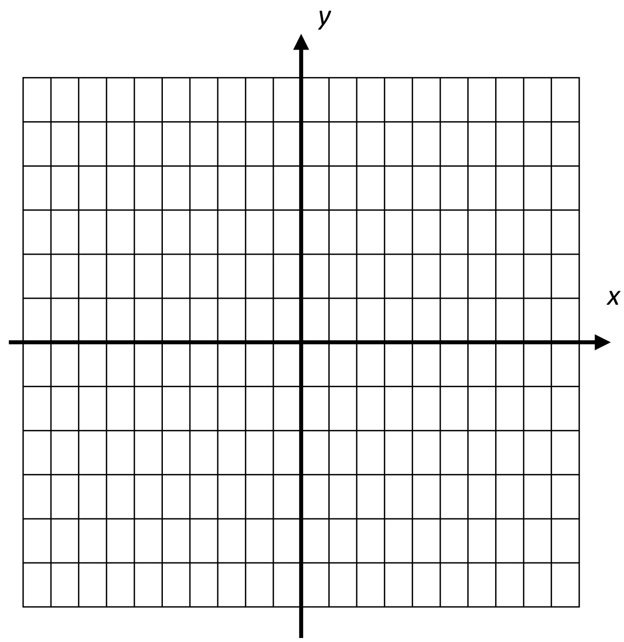
The cube root also gives rise to the **cube root function**. Like the square root function, its basic graph is relatively easy to construct.

***Exercise* #4:** Consider the basic cubic function $y = \sqrt[3]{x}$.

(a) Fill out the table of values below without the use of your calculator.

x	-8	-1	0	1	8
y					

(b) Plot its graph on the grid provided below.



(c) Using your calculator, produce a graph to verify what you found in part (b).

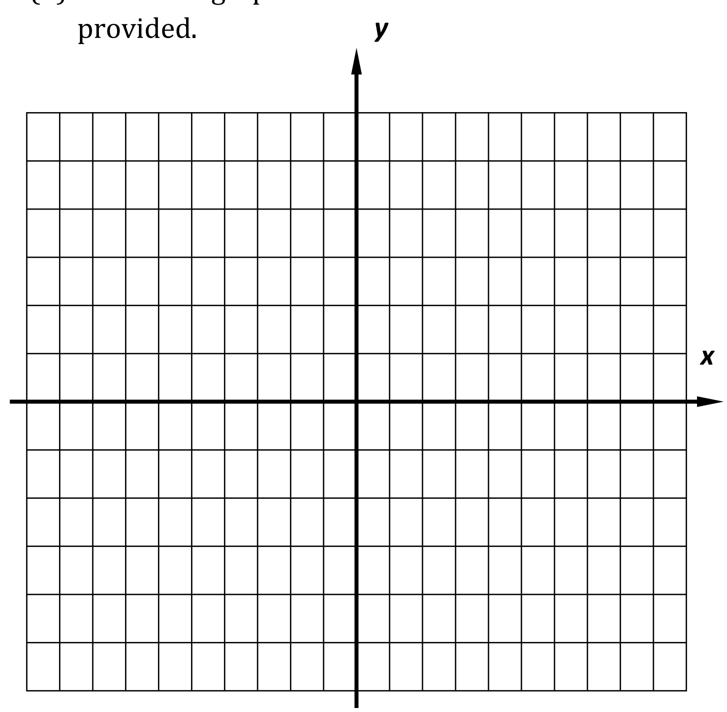
Just like with all other functions, cube root graphs can be **transformed** in a variety of ways. Let's see if our **shifting pattern** continues to hold with cube roots.

***Exercise* #5:** Consider the function $f(x) = \sqrt[3]{x+2} - 4$.

(a) Use your calculator to create a table of values that can be plotted. Show your table below.

x					
y					

(b) Create a graph of this function on the axes provided.



(c) Describe how the graph you drew in Exercise #4 was shifted to produce this graph?

Name: ______________________________ Date: ______________ Period: _____
Algebra Cube Roots 9G HW

1. Find each of the following cube roots without the use of your calculator. Justify your answer based on a multiplication statement.

(a) $\sqrt[3]{8}$ (b) $\sqrt[3]{-1}$ (c) $\sqrt[3]{125}$ (d) $\sqrt[3]{0}$

(e) $\sqrt[3]{-8}$ (f) $\sqrt[3]{27}$ (g) $\sqrt[3]{\frac{1}{64}}$ (h) $\sqrt[3]{-\frac{1}{1000}}$

2. Use your calculator to find the following cube roots by trial and error. Justify your answers using a multiplication statement.

(a) $\sqrt[3]{512}$ (b) $\sqrt[3]{-2197}$ (c) $\sqrt[3]{9261}$ (d) $\sqrt[3]{-15,625}$

3. The cube root function is the inverse of the cubing (x^3) function. Just as we can solve certain quadratic equations by using square roots, we can solve certain cubic equations by using cube roots. Solve each of the following in the form required. Use your calculator on (b) to find the cube root.

(a) $2x^3-1=53$ (Solve exactly) (b) $\frac{x^3}{8}-3=7$ (Solve to nearest tenth)

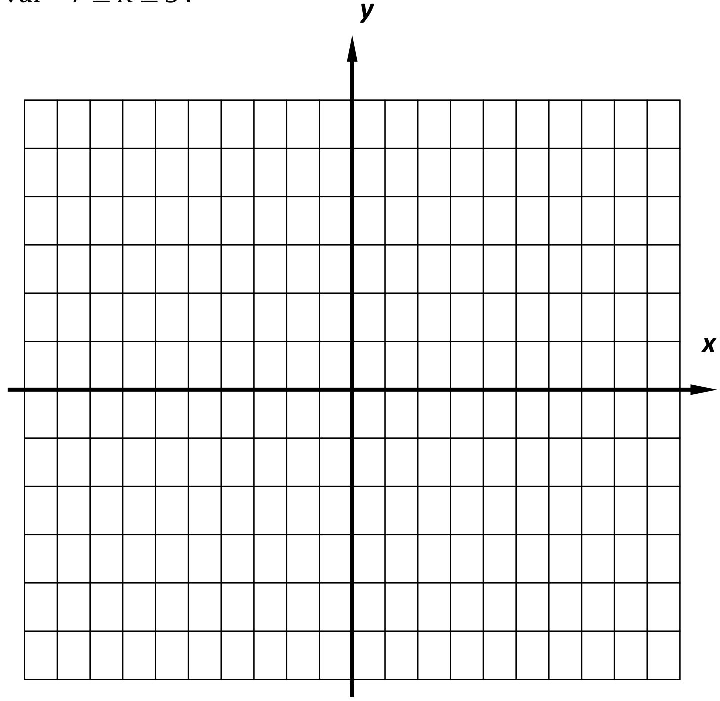
4. If $g(x)=5\sqrt[3]{x+7}-4$, then which of the following is the value of $g(57)$?

(1) 19 (3) 16
(2) 11 (4) 25

5. Consider the function $f(x) = \sqrt[3]{x-1} + 2$ over the interval $-7 \le x \le 9$.

(a) Graph $f(x)$ over this domain interval only.

Must include a table



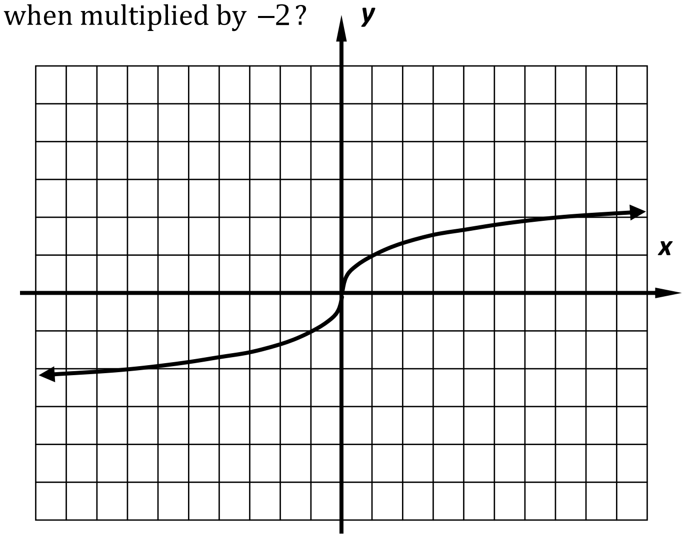
(b) State the range of the function over this interval.

(c) Recall that the average rate of change over the interval $a \le x \le b$ is calculated by $\dfrac{f(b) - f(a)}{b - a}$. Find the average rate of change of $f(x)$ over the intervals below:

(i) $2 \le x \le 9$ (ii) $0 \le x \le 1$ (iii) $-7 \le x \le 9$

6. The graph of $y = \sqrt[3]{x}$ is shown below. On the same set of axes, graph $f(x) = -2\sqrt[3]{x}$. Fill out the table below to help with your graph. What happened to the graph of $y = \sqrt[3]{x}$ when multiplied by -2?

x	-8	-1	0	1	8
$f(x) = -2\sqrt[3]{x}$					



6. Explain why it is not possible to find the square root of a negative number but it is possible to find the cube root of a negative number. Give examples to support your explanation.

Review Section:

7. What are the zeros of the following function, $f(x) = 2x^2 - 4x - 6$?

8. A construction company uses the function $f(p)$, where p is the number of people working on a project, to model the amount of money it spends to complete a project. A reasonable domain for this function would be:

(1) positive integers
(2) positive real numbers
(3) both positive and negative integers
(4) both positive and negative real numbers.

Name: *Homework Answers* Date: ______ Period: ____

Algebra Cubic Roots 9G HW

1.) (a) 2 (b) -1 (c) 5 (d) -2
(e) -2 (f) 3 (g) $\frac{1}{4}$ (h) $-\frac{1}{10}$

2.) (a) 8 (b) -13 (c) 21 (d) -25

3.) (a) 3 (b) 4.3

4.) (3)

5.) (a) Graph and Table (b) $0 \leq y \leq 4$ (c) $m = \frac{1}{7}, m = 1, m = \frac{1}{4}$

6.) Graph and Table; it flipped over x-axis and was stretched by 2.

6.) Proper explanation.

7.) $x = \{-1,3\}$

8.) (1)