Chapter 1

Essentials of Algebra

Chapter 1: Essential Algebra Concepts
Lesson 1:
Variables, Terms, and Expressions

Basic Definitions:

Variable: A quantity that is represented by a letter or symbol that is _____________, _____________, or can change within the context of a problem.

Terms: A single number or combination of numbers and variables using exclusively multiplication or division. This definition will expand when we introduce higher-level functions.

Expression: A combination of _____________using _____________and _____________.

Like Terms: Two or more terms that have the same _____________raised to the same _____________. In like terms, only the coefficients (the multiplying numbers) can differ.

Algebraic Expressions: are just _____________of constants and variables using the typical operations of addition, subtraction, multiplication, and division along with exponents and roots (square roots, cube roots, etcetera).

Examples:

***Exercise* #1:** Consider the expression $2x^2 + 3x - 7$.

(a) How many terms does this expression contain?

(b) Evaluate this expression, when $x=-3$. Show your calculations.

(c) What is the sum of this expression with the expression $5x^2 - 12x + 2$?

***Exercise* #2:** Consider the algebraic expression $4x^2 + 1$.

(a) Describe the operations occurring within this expression and the order in which they occur.

(b) Evaluate the expression x = -3.

***Exercise* #3:** Consider the more complex algebraic expression (known as a rational expression): $\frac{4x+3}{x^3-7}$.

(a) Find the value of the expression when x = 3. Show all of your steps. Final answer must be in simplest form.

(b) Michael entered the following expression into his calculator.

4(3) + 3/3^3 – 7

Would he get the correct answer? Why or why not.

Exercise #4: Given the expression $\sqrt{25 - x^2}$.
(a) Can you evaluate the expression for x = 13? Why not?

(b) Louis thinks that the square root operation distributes over the subtraction. In other words, he believes the following equation is an identity:

$$\sqrt{25 - x^2} = 5 - x$$

Show that this is ***not*** an identity.

Linear Equations:

Linear equations are an important topic in algebra. These types of equations will be used in different topics throughout the year.

In mathematics, both a linear and non linear system are said to be ___________________ if there is at least one solution to the problem. An ___________________ is when both sides of the equation work out to be exactly the same - there are an ___________________ amount of solutions. An equation is said to be ___________________ if there are no set of values that will work for the problem - this means that there is ___________________.

Examples: For the following examples, solve the linear equation. If the system is ***inconsistent*** or an ***identity,*** state that and justify your answer.

***Exercise #5*:** Solve the following equations.

(a) $3x + 5 = 26$

(b) $6x - 2(x + 4) = 3(x + 2) + x - 5$

Exercise #6: Answer both parts (a) and (b).

(a) $8x – 7 = 4x – 5$

(b) $2x - 6 + x - 1 = 3(x - 3) + 2$

***Exercise #7*:** Which of the following equations are identities, which are inconsistent, and which are neither? Solve.

(a) $8x - 2(x + 3) = 5(x - 1) + x$

(b) $\frac{x+8}{2} = -6$

(c) $2x + 8 - (x - 7) = 2(2x - 3)$ (d) $\frac{4x+2}{2} + 8 = 2x + 9$

Exercise #8: Given that (5x + 3) - (2x + 1) = *a*x + *b* is an algebraic identity in x, what are the values of *a* and *b*?

Exercise #9: Given that (x + 3) - 3(2x + 4) = *a*x + *b* is an algebraic identity in x, what are the values of *a* and *b*?

Chapter 1: Essential Algebra Concepts
Lesson 1: Homework
Variables, Terms, and Expressions

Fluency:

1.) Which of the following expressions has the greatest value when x = 5? Show how you arrived at your choice.

$2x^2 + 7$ $\quad\quad$ $\frac{x^3-5}{3}$ $\quad\quad$ $\frac{10x-2}{x-3}$

2.) If x = 5 and y = -2, then $\frac{x+y}{x^2-y^2}$ is

(1) $\frac{1}{7}$ $\quad$ (2) $\frac{13}{3}$ $\quad$ (3) $\frac{3}{29}$ $\quad$ (4) $\frac{7}{19}$

3.) Solve the following linear equations. Please state if the equation is an identity or inconsistent. Reduce any non-integer answers to fractions in simplest form.

(a) $7x + 5 = 2x - 35$ $\quad\quad$ (b) $\frac{x}{3} - 7 = -5$

(c) $4x - (2x - 1) = x + 5 + x - 6$

(d) $\frac{2x+5}{6} = \frac{x}{18}$

(e) $\frac{10x-4}{2} + 7 = 5(x + 1)$

4.) Which of the following is equivalent to the expression 2(x – 6) + 4(2x + 1) + 3?

(1) 8(x – 2)   (2) 5(2x – 1)   (3) 4(2x + 3)   (4) 10(x – 1)

Applications:

5.) The revenue, in dollars, that eMath Instruction makes off its videos in a given day depends on how many views they receive. If *x* represents the number of views, in hundreds, then the profit can be found with the expression:

$$\frac{1}{2}x^2 + 6x - 10$$

How much revenue would they make if their videos were viewed 600 times?

6.) When finding the intersection of two lines, we first "set the linear equations equal" to each other. Find the intersection point of the two lines whose equations are shown below. Be sure to find **both the x and y coordinates**!

y = 5x + 1 and y = 2x – 11

7.) Explain why you ***cannot*** find the intersection points of the two shown below. Use algebra to support your answer.

Y = 4x + 1 and y = 4x + 10

8.) Given that $(2x - 5) + 7(2x - 3) = ax + b$ is an algebraic identity in x, what are the values of a and b?

Chapter 1: Essential Algebra Concepts
Lesson 2
Basic Exponent Rules

Laws of Exponents:

Review: The exponent of a number say how many ____________ to use that number in ____________. For example: x^2 = ________

Properties:

Multiplication: When the *bases* are the same, ____________ the exponents.

$(x^a)(x^b)$ = ____________

Example: $(x^4)(x^3)$ =

Division: When the *bases* are the same, ______________ the exponents.

$\frac{x^a}{x^b}=$ _______________

Example: $\frac{x^6}{x^2}=$ _______________

Power: When you have an exponent raised to a power, you _______________the exponent and the power.

$(x^a)^b=$_______________

Example: $(w^2x^7)^5$ = _______________

Zero as an exponent: Anything to the zero power is *always* equal to 1!

Warning! ***The power rule does not apply when you have a sum or difference within the parentheses. Exponents, unlike multiplication, do not "distribute" over addition.***

Examples:

1.) Write $\frac{4x^3y^4}{2x^5y^2}$ without a denominator.

Example #2: Simplify the following.

(a) $(a^2)(a^6)$

(b) $(2b^3)(-6b^7)$

(c) $(-2c^5)^3$

(d) $\frac{-5y^5z^7}{15y^2z^5}$

(e) $\frac{30xy^6}{-5y^2}$

(f) $(5x^8)^2$

(g) $\left(\frac{3}{2}x^2\right)(4x^3)^2$

Negative Exponents

Normally, in math, we should not use negative exponents. A negative exponent just means that the base is on the _______________ side of the fraction line, so we would need to move the base to the _______________ side.

Example #3: Write all answers using only positive exponents.

(a) $2x^{-4}$ (b) $(-5x^3)(12x^{-4})$ (c) $(3y)^{-3}$

(d) $\frac{12x^{-2}}{15x^3}$ (e) $(4x^{-5})^2$ (f) $\left(\frac{2x^2}{3y^4}\right)^{-3}$

Example #3: What is the value of 3^{-2}?

(1) $\frac{1}{9}$ (2) $-\frac{1}{9}$ (3) 9 (4) -9

Example #4: If a = 3 and b = -2, what is the value of the expression $\frac{a^{-2}}{b^{-3}}$?

(1) $-\frac{9}{8}$ (2) -1 (3) $-\frac{8}{9}$ (4) $\frac{8}{9}$

Example #5: Use the distributive property to multiply together the following monomials and polynomials.

(a) $2x(5x + 3)$

(b) $5x^3(2x^2 - 3x + 6)$

(c) $-7x^2(x^2 - 2x + 3)$

(d) $3x^2y^4(2x^2y + xy^2 - 4y^3)$

Example #6: Fill in the missing portions so the equation is an *identity*.

(a) $8x^2 - 12x = 4x($ __________________________ $)$

(b) $7x^4 - 21x^3 - 28x^2 = 7x^2($ ____________________________ $)$

(c) $4x^2(x - 2) - 9(x - 2) = (x - 2)($ ____________________________ $)$

Example #7: Rewrite $(3^{-2})^4$ as $\frac{1}{3^8}$. Show all steps and justify your answer.

Chapter 1: Essential Algebra Concepts
Lesson 2: Homework
Basic Exponent Rules

Fluency:

1.) Simplify each of the following expressions so there are no negative exponents.

(a) $(3x^2)(10x^4)$

(b) $(4x^2y^{-3})\left(\frac{1}{16}x^5y^{-3}\right)$

(c) $\left(\frac{2}{3}x^4\right)(12x^{-5})$

(d) $(6x^{-2})^{-3}$

(e) $(2x^2y)(5x)^{-2}(-6x^{-3})$

(f) $\frac{-12x^{-2}y^3}{21x^3y^{-2}}$

2.) The exponential expression $\left(\frac{1}{8}\right)^4$ is equivalent to which of the following?

(1) 4^{-8} (2) 2^{-12} (3) 8^{-2} (4) 32^{-1}

3.) Which expression is equivalent to $(3x^2)^{-1}$?

(1) $\frac{1}{3x^2}$ (2) $-3x^2$ (3) $\frac{1}{9x^2}$ (4) $-9x^2$

4.) Use the distributive property to write each of the following products as polynomials.

(a) $4x(5x + 2)$ (b) $6x(x^2 - 4x + 8)$

(c) $-10x^2(2x^2 + x - 8)$ (d) $8x^2y^2(x^3 - 2x^2y + 5xy^2 - y^3)$

5.) Fill in the missing part of each product in order to make the equation into an identity.

(a) $10x^5 - 35x^3 = 5x^3($ __________________)

(b) $-8x^3y + 2x^2y^2 - 10xy^3 = -2xy$ (__________________)

(c) $x^2 (x - 3) - (x - 3) = (x - 3)$ (__________________)

Chapter 1: Essential Algebra Concepts
Lesson 3
Multiplying Polynomials

Multiplying Polynomials

Recall: A **polynomial** is an *algebraic expression that is the* _______________ *or* _______________ *of two or more* ________________________. Whenever we are using polynomials, it is important to write them in ***descending*** order.

Multiplying polynomials is similar to multiplying monomials. We must _______________ *the coefficients* and _______________ *the exponents*. Whenever we multiply monomials we use double (or triple) _______________.

Example 1: Determine the product of (3x + 2) and (2x + 5).

***Example* #2:** Find the product of the binomial (4x + 3) with the trinomial ($2x^2 - 5x - 3$).

Example #3: Find each product below.

(a) (8 – y)(8 + y) (b) (2x + 7)(6 – 5x)

(c) $(y^2 - 8y + 2)(y^2 + 9y - 2)$

(d) $(2xy + 3)(5xy - 3)$

(e) $\left(\frac{1}{2}k + 2\right)\left(\frac{3}{4}k - 4\right)$

(f) $(3k^2 + 12k - 3)(k - 10)$

Example #4: Expand the expressions below.

(a) $(2x - 1)^2$

(b) $(3x + 2)^3$

Example #5: A square of unknown side length *x* centimeters has one side length increased by 5 and the other side length decreased by 3 centimeters to create a rectangle.

(a) Write an expression to represent the dimensions of the rectangle.

(b) Determine the area of the rectangle as a polynomial function.

(c) If the original square had a side length of 7 centimeters, what is the area of the new rectangle?

Example #6: What is the product of $\left(\frac{x}{4}-\frac{1}{3}\right)$ and $\left(\frac{x}{4}+\frac{1}{3}\right)$?

(1) $\frac{x^2}{8}-\frac{1}{9}$ (2) $\frac{x^2}{16}-\frac{1}{9}$ (3) $\frac{x^2}{8}-\frac{x}{6}-\frac{1}{9}$ (4) $\frac{x^2}{16}-\frac{x}{6}-\frac{1}{9}$

Example #7: Prove that the expression below is an identity:

$(x + 5)(x + 8) - (x + 3)(x - 2) = 12x + 46$

Chapter 1: Essential Algebra Concepts
Lesson 3: Homework
Multiplying Polynomials

Fluency:

1.) Multiply the following polynomials and write as a single polynomial expression.

(a) $(x + 5)(x + 8)$

(b) $(x^2 - 4)(x^2 - 10)$

(c) $(2x^3 + 1)(5x^3 + 4)$

(d) $(2x - 3)(4x^2 + 5x - 7)$

(e) $(2y^2 + 7y - 1)(3y^2 + 4y + 2)$

(f) $\left(\frac{1}{2}m - 2\right)\left(\frac{3}{4}m^2 + \frac{1}{2}m - 12\right)$

Applications:

2.) A square of an unknown side length x has one length increased by 4 inches and the other increased by 7 inches.

(a) Find the dimensions of the rectangle that you created.

(b) Find the area of the rectangle.

(c) If the original square had a side length of $x = 2$ inches, then what is the area of the rectangle? Show how you arrived at your answer.

3.) Given the expression $(x - 8)(x + 4)$

(a) For what values of x will this expression be equal to zero? Show how you arrived at your answer.

(b) Write this product as an equivalent trinomial.

Regents Question:

4.) Algebraically determine the values of *h* and *k* to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

Chapter 1: Essential Algebra Concepts

Lesson 4

Rational Exponents

Rational (Fractional) Exponents

$x^{\frac{P}{R}}$ The exponent is now a fraction.

P = ______________

R = ______________

R = tells you what __________ to take

P = tells you what _____________ to ______________ it to

____

There are multiple ways of writing fractional powers. We should be familiar with them all.

When you are dealing with a radical expression, you can convert it to an expression containing a rational (fractional) power. This conversion may make the problem easier to solve.

Example #1: Rewrite each of the following using roots instead of fractional exponents; then evaluate.

(a) $125^{\frac{1}{3}}$

(b) $16^{1/4}$

(c) $9^{-1/2}$

(d) $32^{-1/5}$

Example #2: Given the function $f(x) = 2(x + 8)^{2/3}$, which of the following represents its y-intercept?

(1) 4 (2) 8 (3) 0 (4) 16

Example #3: Evaluate the following expressions. Show your steps.

(a) $\frac{3^{\frac{1}{3}}}{3^{-\frac{2}{3}}}$

(b) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

(c) $\left(\frac{3^0}{27^{\frac{2}{3}}}\right)^{-1}$

(d) $4^{\frac{1}{2}} \cdot 2^3$

Example #4: What is the value of $4x^{1/2} + x^0 + x^{-1/4}$ when $x = 16$?

Example #5: What is the value of the expression $2x^{-1/3}$ when x = 8?

Example #6: Simplify each expression so you do not have any negative exponents or radicals.

(a) $\left(2x^{-\frac{2}{3}}y^{\frac{1}{2}}\right)\left(\frac{5}{2}x^{\frac{5}{3}}y^{\frac{-7}{2}}\right)$

(b) $\frac{\sqrt[3]{(8m^2n^6)}}{12m^{\frac{4}{3}}n^3}$

Chapter 1: Essential Algebra Concepts
Lesson 4: Homework
Rational Exponents

Fluency:

1.) Rewrite the following as equivalent roots, then evaluate.

(a) $36^{-1/2}$ (b) $4^{3/2}$ (c) $81^{5/4}$

(d) $4^{-5/2}$ (e) $128^{3/7}$ (f) $625^{-3/4}$

2.) Given the function $f(x) = 5(x + 4)^{3/2}$, which of the following represents its y-intercept?

(1) 40 (2) 20 (3) 4 (4) 30

3.) Which of the following is *not* equivalent to $16^{3/2}$?

(1) $\sqrt{4096}$ (2) 8^3 (3) 64 (4) $\sqrt{16^3}$

4.) Which of the following is equivalent to $x^{-1/2}$?

(1) $-\frac{1}{2}x$ (2) $-\sqrt{x}$ (3) $\frac{1}{\sqrt{x}}$ (4) $-\frac{1}{2x}$

5.) Which expression is equivalent to $(9x^2y^6)^{-\frac{1}{2}}$?

(1) $\frac{1}{3xy^3}$ (2) $3xy^3$ (3) $\frac{3}{xy^3}$ (5) $\frac{xy^3}{3}$

6.) Simplify the expression $(m^6)^{-\frac{2}{3}}$ and write your answer using a positive exponent.

Reasoning:

7.) Rachel claims that the square root of a cube root is a sixth root. Is she correct? Explain your reasoning.