

Chapter 11A: Trigonometry Unit 1
Lesson 1: Radian Measures

Radians are another way that we can measure angles. One reason we use radians over degrees is because it is more of a natural measure for dividing up circles.

A **radian** is defined as the *unit of measure of a central angle of a circle that intercepts an arc equal in length to the radius of a circle.*

It is important that we know how to go back and forth between radians and degrees.

To Convert Radians to Degrees:

To Convert Degrees to Radians:

Example #1: Convert each of the following into radian measure:

(a) -50°

(b) 240°

(c) 540°

(d) -330°

(e) 105°

(f) -225°

Example #2: Convert each of the following radian into degree measure:

(a) $\frac{2\pi}{5}$

(b) $\frac{6\pi}{7}$

(c) $\frac{4\pi}{3}$

(d) $\frac{7\pi}{8}$

Example #3: Convert each of the following radian angles, which aren't in terms of pi, into degrees. Round your answers to the nearest degree.

(a) $\theta = 5.8$

(b) $\theta = 4.2$

(c) $\theta = -2.5$

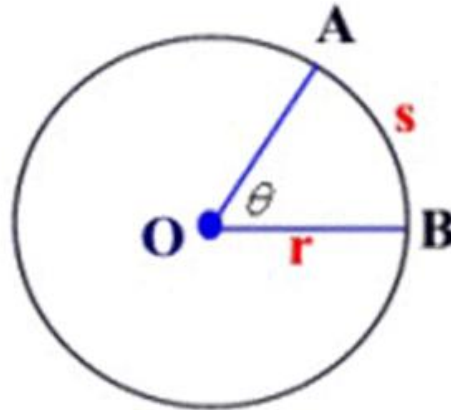
(d) $\theta = 1.0$

Arc Measures:

We can now determine the length of an arc. If θ is the radian measure of the central angle, r is the radius and S is the length of the intercepted arc, then:

$$S = \theta r$$

arc angle
(in radians) radius



Example #4: A wedge-shaped piece is cut from a circular pizza. The radius of the pizza is 6 inches. The rounded edge of the crust of the piece measures 4.2 inches. What is the number of radians, to the nearest tenth of a radian, in the angle at the pointed end of the piece of pizza?

Example #5: What is the radian measure of the smaller angle formed by the hands of a clock at 8 o'clock?

Exercise #6: If a pendulum swings through an angle of 0.55 radians, what distance does its tip travel if it has a length of 8 feet?

Exercise #7: A circle is drawn to represent a pizza with a 12 inch diameter. The circle is cut into eight congruent pieces. What is the length of the outer edge of any one piece of this circle?

Chapter 11A: Trigonometry Unit 1
Lesson 1: Radian Measures
Homework

1.) Convert each of the following common degree angles to angles in radians. Express your answers in exact terms of pi.

(a) 30°

(b) 45°

(c) 60°

(d) 180°

(e) 300°

(f) 135°

(g) 270°

(h) 330°

2.) Convert each of the following angles given in radians into an equivalent measure in degrees. Your answers will be integers.

(a) $\frac{2\pi}{3}$

(b) $-\frac{\pi}{2}$

(c) $\frac{11\pi}{4}$

(d) $-\frac{4\pi}{3}$

3.) What is the number of degrees in an angle whose radian measure is $\frac{11\pi}{12}$?

(1) 150

(2) 165

(3) 330

(4) 518

4.) What is the radian measure of the smaller angle formed by the hands of a clock at 7 o'clock?

(1) $\frac{\pi}{2}$

(2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$

(4) $\frac{7\pi}{6}$

5.) A dog is attached to a 10 foot leash. He travels around an arc that has a length of 25 feet. Which of the following represents the radian angle he has rotated through?

(1) 5

(2) 7.5π

(3) 2.5

(4) 1.25π

6.) A wheel whose diameter is 3 feet rolls a distance of 45 feet without slipping. Through what radian angle did the wheel rotate?

(1) 30

(2) 25

(3) 30π

(4) 12π

7.) The distance from the center of a Ferris wheel to a person who is riding is 38 feet. What distance does a person travel if the Ferris wheel rotates through an angle of 4.25 radians?

(1) 80.75 feet

(2) 42.5 feet

(3) 507 feet

(4) 161.5 feet

8.) A golfer swings a club about a pivot point. If the head of the club travels a distance of 26 feet and rotates through an angle of 5 radians, which of the following gives the distance the club head is from the pivot point?

(1) 1.7 feet

(2) 2.6 feet

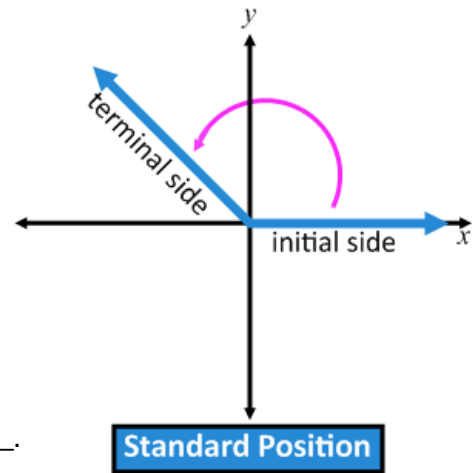
(3) 5.2 feet

(4) 7.2 feet

Chapter 11A: Trigonometry Unit 1

Lesson 2: Coterminal Angles

Standard Position: An angle is in **standard position** if its vertex is located at the origin and one ray is on the positive x-axis. The ray on the x-axis is called the **initial side** and the other ray is called the **terminal side**.



Angles on the Coordinate Plane:

Recall: Since there are ____ in a full rotation, each quadrant contains ____.

Quadrant I contains angles that measure *between* _____° and _____°

Quadrant II contains angles that measure *between* _____° and _____°

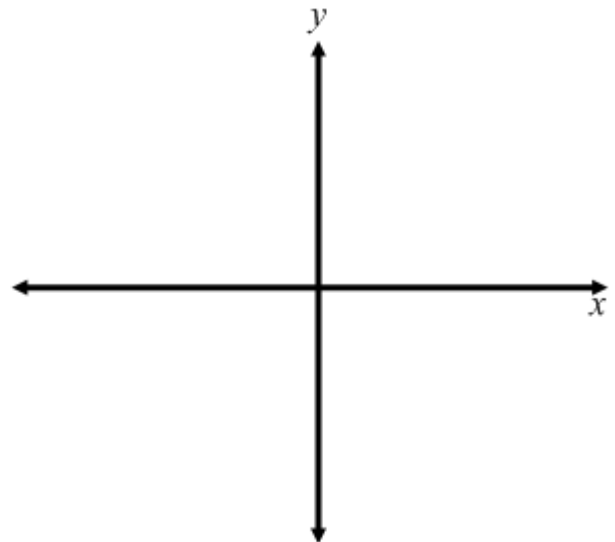
Quadrant III contains angles that measure *between* _____° and _____°

Quadrant IV contains angles that measure *between* _____° and _____°

Notice: Each of these definitions is for angles that fall *inside* the quadrants.

If the terminal side of an angle lies "**on**" the axes such as **0°, 90°, 180°, 270°, 360°** it is called a **quadrantal angle**.

These angles are not "in" a quadrant, but rather separate the quadrants.



Coterminal Angles:

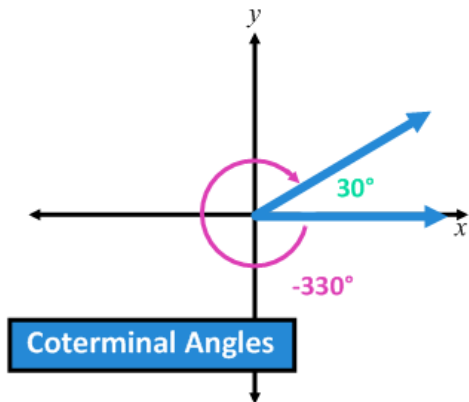
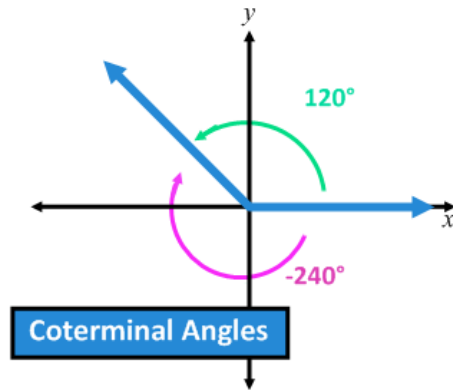
The angle is measured by the amount of rotation from the initial side to the terminal side.

If measured in a **counterclockwise** direction the measurement is **positive**.

If measured in a **clockwise** direction the measurement is **negative**.

(A negative associated with an angle's measure refers to its "direction" of measurement, clockwise.)

If two angles in standard position have the same terminal side, they are called **coterminal** angles.



Coterminal angles are angles in standard position that have a common terminal side. For example, 30° , -330° and 390° are all coterminal.

To find a positive and negative angle coterminal with a given angle, add or subtract any multiple of 360° .

Example 1:

Find a coterminal angle of the -280° angle with a measure between 0° and 360° . Draw a rotation diagram and then state what quadrant the angle lies in.

Example 2:

Find a coterminal angle of the 500° angle with a measure between 0° and 360° . Draw a rotation diagram and then state what quadrant the angle lies in.

Example 3:

Find a coterminal angle of the $8\pi/3$ radian angle with a measure between 0 radians and 2π radians. Draw a rotation diagram and then state what quadrant the angle lies in.

Example 4:

Find a coterminal angle of the $-11\pi/4$ radian angle with a measure between 0 radians and 2π radians. Draw a rotation diagram and then state what quadrant the angle lies in.

Example #5:

For each of the following angles, given by the Greek letter **theta**, draw a rotation diagram and identify the quadrant that the terminal ray falls in.

(a) $\theta = 145^\circ$

(b) $\theta = 320^\circ$

(c) $\theta = \frac{4\pi}{5} \text{ rad}$

(d) $\theta = -310^\circ$

(e) $\theta = -400^\circ$

(f) $\theta = 460^\circ$

Example #6: In which quadrant would the terminal ray of an angle drawn in standard position fall if the angle measures 860° ?

(1) I

(2) II

(3) III

(4) IV

Example #7: Coterminal angles drawn in standard position will always have measures that differ by an integer multiple of

- (1) 90°
- (2) 360°
- (3) 180°
- (4) 720°

Example #8: An angle drawn in standard position whose radian measure is 2 radians would terminate in which of the following quadrants?

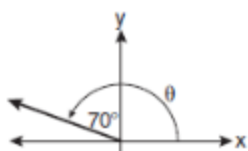
- (1) I
- (2) II
- (3) III
- (4) IV

Example #9: Which of the following would *not* be coterminal with an angle that measures 250° ?

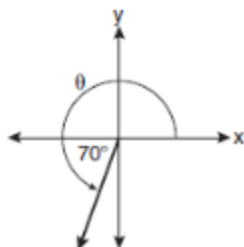
- (1) -110°
- (2) 610°
- (3) 110°
- (4) 970°

Chapter 11A: Trigonometry Unit 1
Lesson 2: Coterminal Angles
Homework

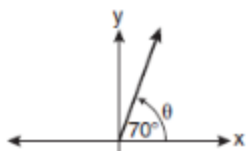
1.) In which graph is θ coterminal with an angle of -70° ?



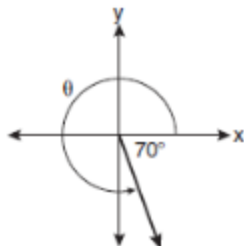
(1)



(3)



(2)



(4)

2.) For each of the following angles, draw a rotation diagram and then state the quadrant the terminal ray of the angles falls within.

(a) $\theta = -370^\circ$

(b) $\theta = -\frac{2\pi}{3}$

(c) $\theta = 97^\circ$

(d) $\theta = -310^\circ$

(e) $\theta = \frac{15\pi}{6}$

(f) $\theta = 560^\circ$

3.) Give two angles that are coterminal with each of the following angles. Make one of the coterminal angles positive and one negative.

(a) $\theta = 105^\circ$

(b) $\theta = 220^\circ$

(c) $\theta = 80^\circ$

4.) When drawn in standard position, which of the following angles is coterminal to one that measures 130° ?

(1) 430°

(3) 850°

(2) -70°

(4) 730°

5.) The angle 1010° lies in Quadrant

(1) I

(3) III

(2) II

(4) IV

6.) Which angle is coterminal with $-\frac{\pi}{4}$?

(1) 4π

(3) $\frac{3\pi}{4}$

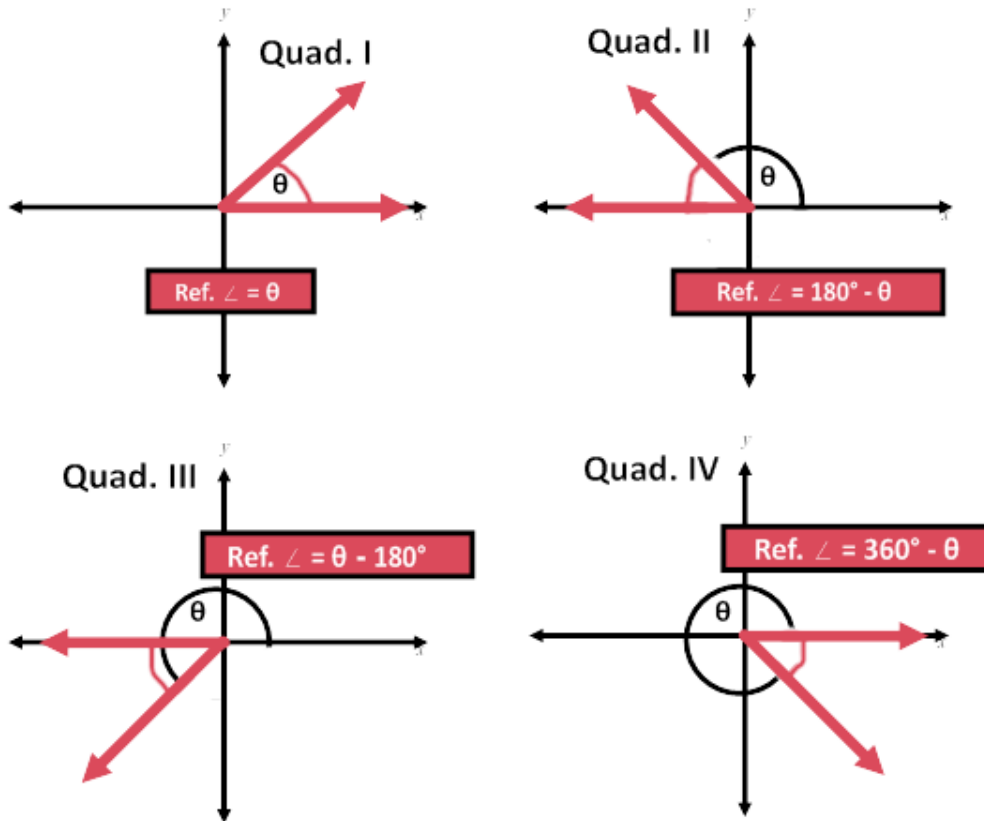
(2) $\frac{\pi}{4}$

(4) $\frac{7\pi}{4}$

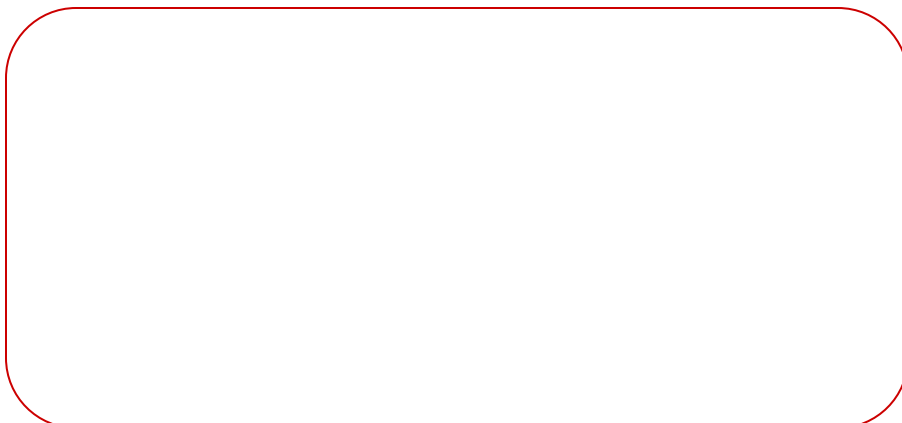
Chapter 11A: Trigonometry Unit 1
Lesson 3: Reference Angles

Reference Angles:

Associated with every angle drawn in standard position (except quadrantal angles) there is another angle called the **reference angle**. The reference angle is the acute angle formed by the **terminal side** of the given angle and the **x-axis**. Reference angles may appear in all four quadrants. Angles in quadrant I are their own reference angles.



REMEMBER: The reference angle is measured from the terminal side of the original "to" the x-axis. (**NEVER the y-axis**).

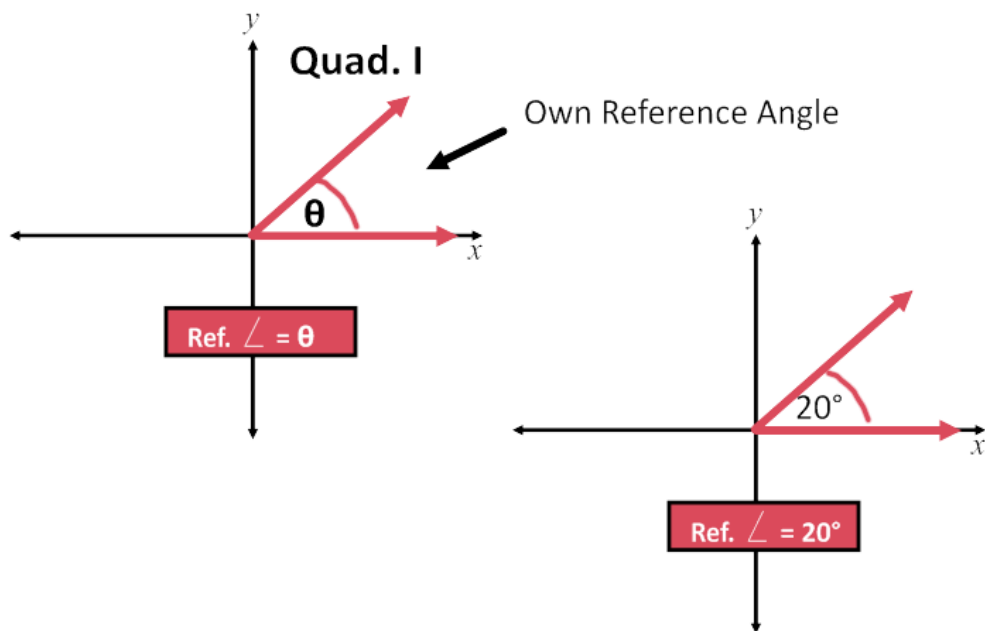


When Do We use Reference Angles?

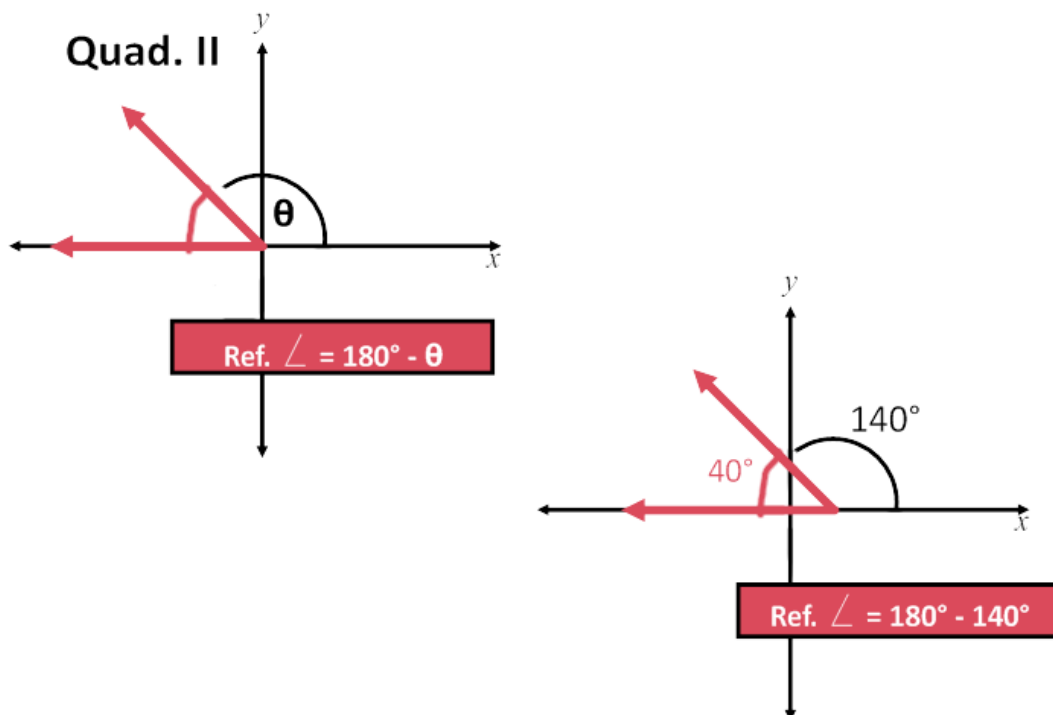
Sometimes we are asked to express a function of an angle that is greater than 90° as a function of a positive acute angle (an angle greater than 0 but less than 90°). We will use reference angles to accomplish this.

The reference angle is the acute angle formed by the terminal side of the given angle and the x-axis. (never the y-axis)

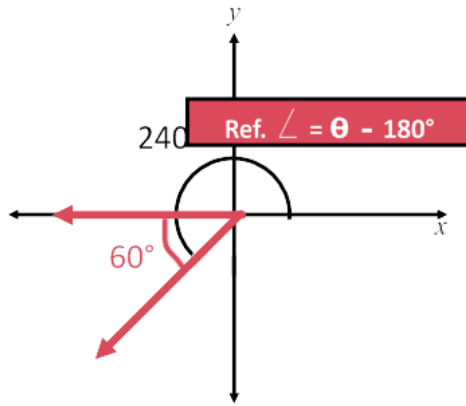
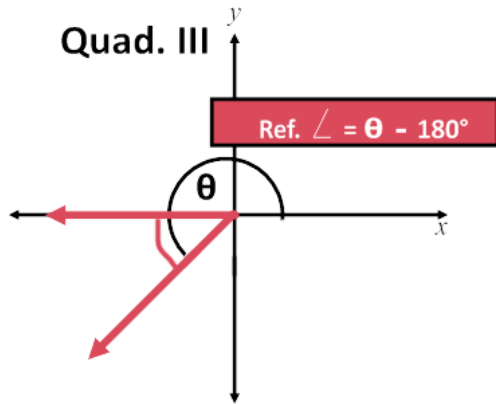
Quadrant I:



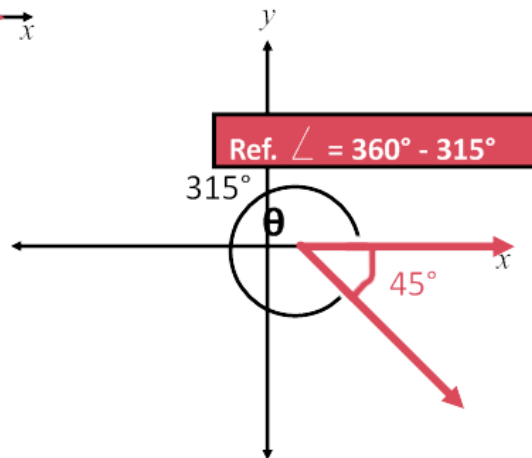
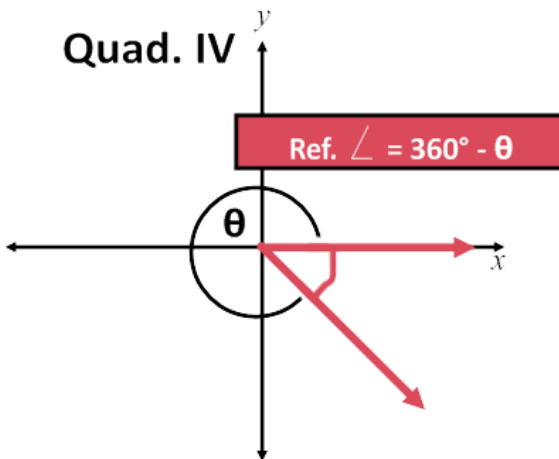
Quadrant II:



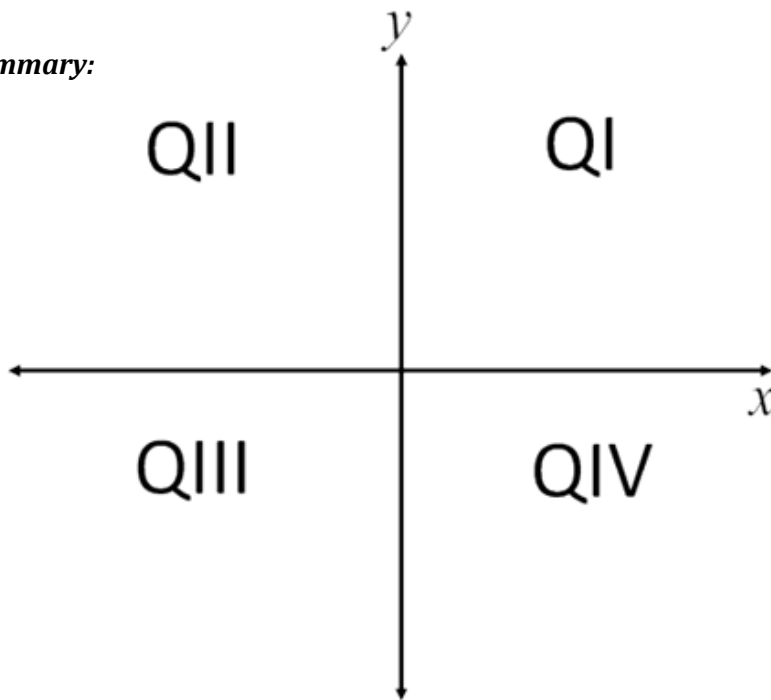
Quadrant III:



Quadrant IV:



Reference Angle Summary:

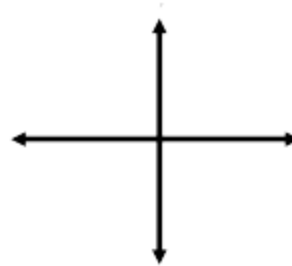
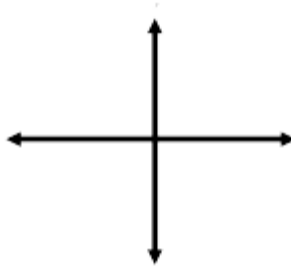
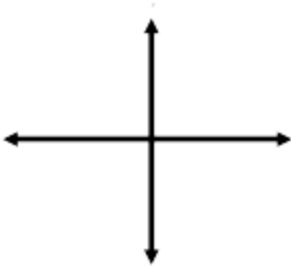


Example 1: Draw the given angle in standard position, then find the reference angle.

(a) 140°

(b) 290°

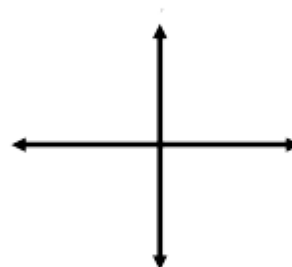
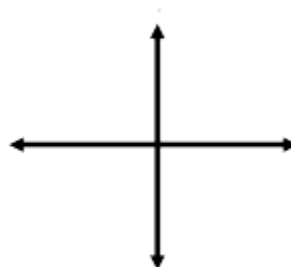
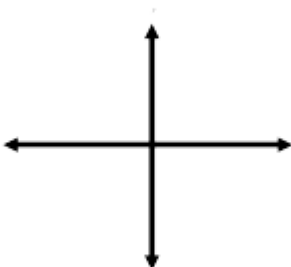
(c) $\frac{3\pi}{4}$ rad



(d) -340°

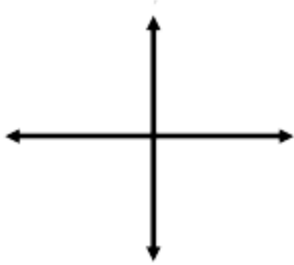
(e) 560°

(f) $-\frac{7\pi}{12}$ rad

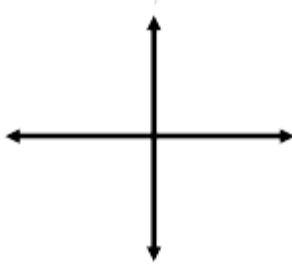


Example 2: For each of the following angles, beta, draw a rotation diagram and then state beta's reference angle, β_r .

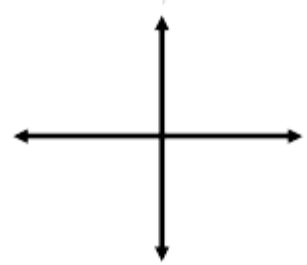
(a) $\beta = 160^\circ$



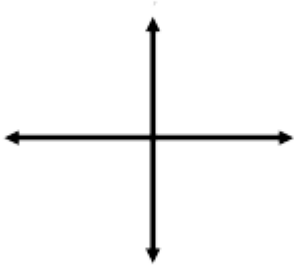
(b) $\beta = 605^\circ$



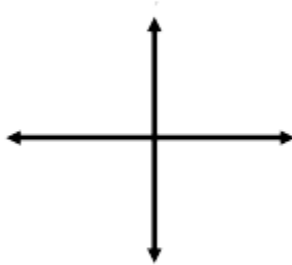
(c) $\beta = -280^\circ$



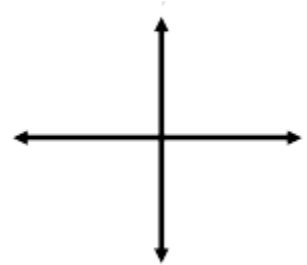
(d) $\beta = \frac{5\pi}{6} \text{ rad}$



(e) $\beta = -\frac{13\pi}{12} \text{ rad}$



(f) $\beta = \frac{11\pi}{4} \text{ rad}$



Example 3: Which of the following would *not* have a reference angle equal to 50° ?

(1) 130°

(3) 490°

(2) 220°

(4) 230°

Chapter 11A: Trigonometry Unit 1
Lesson 3: Reference Angles
Homework

1.) For each of the following angles, draw a rotation diagram and determine the reference angle.

(a) $\alpha = 245^\circ$

(b) $\alpha = 290^\circ$

(c) $\alpha = 130^\circ$

(d) $\alpha = -242^\circ$

(e) $\alpha = 475^\circ$

(f) $\alpha = -432^\circ$

2.) Which of the following angles would *not* have a reference angle equal to 30° ?

(1) 210°

(2) -330°

(3) 120°

(4) -30°

3.) Draw a rotation diagram for each of the following radian angles, which are expressed in terms of pi. Then, determine the reference angle for each, also in terms of pi.

(a) $\frac{2\pi}{3}$

(b) $\frac{11\pi}{6}$

(c) $\frac{5\pi}{4}$

4.) First find the angle coterminal between $0^\circ \leq \theta < 360^\circ$, then find the reference angle.

(a) 582°

(b) 624°

(c) -382°

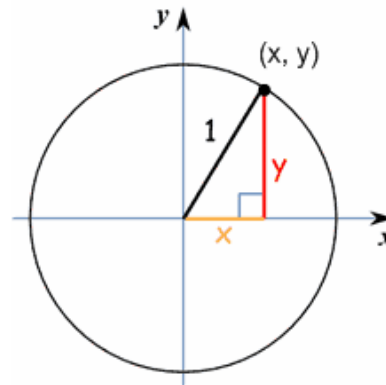
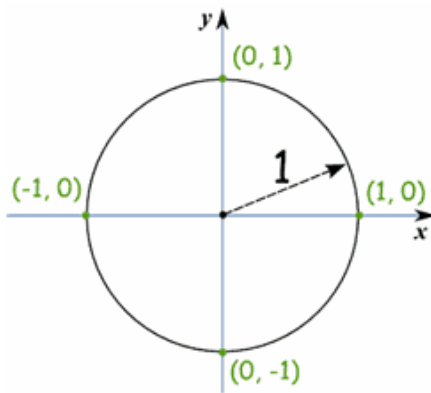
Chapter 11A: Trigonometry Unit 1
Lesson 4: Unit Circle

Recall From Geometry: SOH CAH TOA

Identify the sine, cosine and tangent of the following triangle:

The Unit Circle:

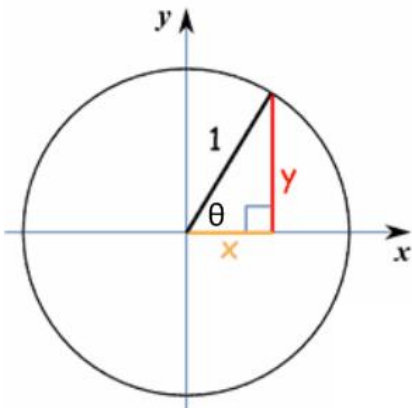
The **unit circle** is a circle with a radius of 1 and a center (0,0).



For a point (x,y) in Quadrant I, the lengths x and y become the legs of a right triangle whose hypotenuse is 1.

*By the Pythagorean Theorem, we have $x^2 + y^2 = 1$
(the equation of the unit circle).*

Examining the Unit Circle:



If we examine angle θ (in standard position) in this **unit circle**, we can see that

$$\cos\theta = \underline{\hspace{2cm}} \quad \sin\theta = \underline{\hspace{2cm}}$$

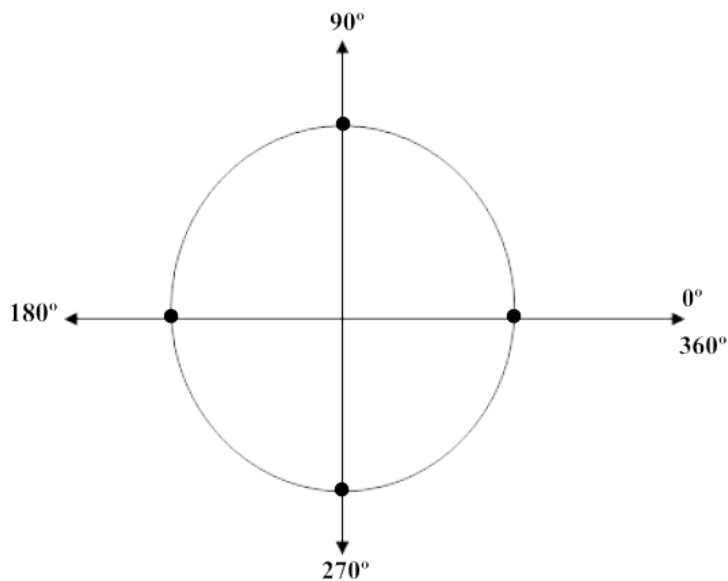
On the unit circle $(x, y) = (\quad , \quad)$

Think: In terms of $\sin\theta$ and $\cos\theta$, what do you think $\tan\theta$ will be equal to? (Recall SOHCAHTOA!)

$\tan\theta =$

Unit Circle with radius length _____

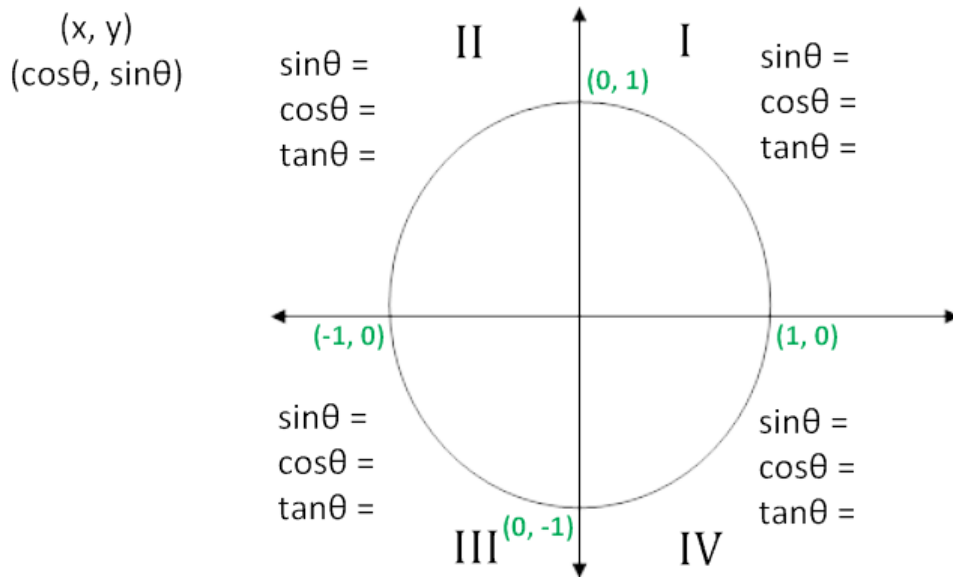
Label the quadrantal angles **and** the **coordinates** of each.



θ	0°	90°	180°	270°	360°
Sin θ					
Cos θ					
Tan θ					

Yes

But, use the unit circle to help organize the details



All Students Take Calculus

As point $P(x, y)$ moves around the unit circle, and θ increases from 0° to 360° , x and y change signs and thus the signs of $\sin\theta$ and $\cos\theta$ also change.

- In the first quadrant, since x and y are both positive numbers, $\sin\theta$ is _____ and $\cos\theta$ is _____.
- In the second quadrant, since x is a negative number and y is a positive number, $\sin\theta$ is _____ and $\cos\theta$ is _____.
- In the third quadrant, since x and y are both negative numbers, $\sin\theta$ is _____ and $\cos\theta$ is _____.
- In the fourth quadrant, since x is a positive number and y is a negative number, $\sin\theta$ is _____ and $\cos\theta$ is _____.

Example #1: Identify the quadrant that each angle θ could lie in?

- 1) Draw a picture
- 2) Label ASTC
- 3) Put a check in the quadrants that satisfy the conditions
- 4) Choose the correct quadrant

(a) $\sin \theta > 0$ and $\cos \theta > 0$

(b) $\sin \theta < 0$ and $\cos \theta > 0$

(c) $\sin \theta < 0$ and $\tan \theta > 0$

(d) $\sin \theta > 0$ and $\cos \theta < 0$

(e) $\tan \theta > 0$ and $\sin \theta > 0$

(f) $\sin \theta < 0$ and $\cos \theta > 0$

(g) $\cos \theta > 0$ and $\tan \theta < 0$

Example #2: The terminal side of $\angle ROP$ is in standard position and intersects the unit circle at point P. If $\angle ROP$ is θ , find:

- (1) $\sin\theta$
- (2) $\cos\theta$
- (3) Quadrant of $\angle ROP$
- (4) $\tan\theta$

if...

(a) $P\left(\frac{5}{13}, \frac{12}{13}\right)$

(b) $P\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

(c) $P(-0.25, -0.23)$

(d) $P(0.6, -0.8)$

Example #3: Using the identity $\sin^2\theta + \cos^2\theta = 1$, find the value of $\sin\theta$, to the *nearest hundredth*, if $\cos\theta$ is -0.8 and θ is in Quadrant II.

Example #4: Using the identity $\sin^2\theta + \cos^2\theta = 1$, find the value of $\tan\theta$, to the *nearest hundredth*, if $\sin\theta$ is -0.4 and θ is in Quadrant III.

Chapter 11A: Trigonometry Unit 1
Lesson 4: Unit Circle
Homework

1.) If $\sin x < 0$ and $\cos x > 0$, then angle x terminates in Quadrant:

(1) I

(2) II

(3) III

(4) IV

2.) If $\sin x < 0$ and $\tan x > 0$ then the measure of x could be

(1) 25

(2) 155

(3) 205

(4) 335

3.) If $\sin A = \frac{4}{5}$ and $\tan A < 0$, in what quadrant does $\angle A$ lie in?

(1) I

(2) II

(3) III

(4) IV

4.) If $\cos x = -\frac{1}{2}$ and $\sin x = -\frac{\sqrt{3}}{2}$ in what quadrant could angle x terminate?

(1) I

(2) II

(3) III

(4) IV

5.) Given a point on the unit circle, using the identity $\sin^2\theta + \cos^2\theta = 1$ to find the missing value.

(a) $x = -\frac{7}{25}$; $y = ?$ in Quadrant III

(b) $y = \frac{40}{41}$; $x = ?$ in Quadrant II

6.) Using the identity $\sin^2\theta + \cos^2\theta = 1$, find the value of $\tan\theta$, to the *nearest hundredth*, if $\sin\theta$ is -0.75 and θ is in Quadrant III.

7.) At which of the following angles is tangent undefined?

(1) $\theta = 0^\circ$

(2) $\theta = 270^\circ$

(3) $\theta = 120^\circ$

(4) $\theta = -180^\circ$

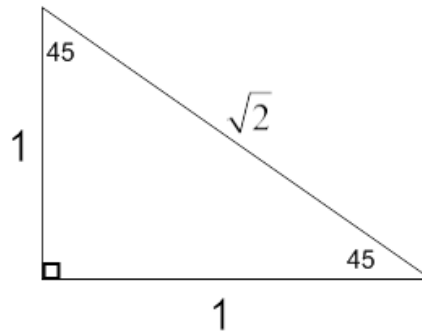
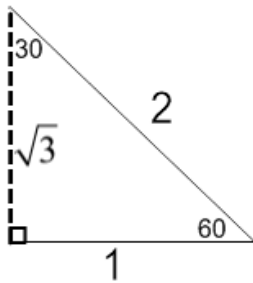
Chapter 11A: Trigonometry Unit 1
Lesson 5: Trig Exact Values

Special Right Triangles

There are two "special" right triangles with which you need to be familiar. The 30 - 60 - 90 triangle and the 45 - 45 - 90 triangle.

The "special" nature of these triangles is their ability to yield exact answers instead of decimal approximations when dealing with trigonometric functions.

These triangles will be used continually throughout your studies of mathematics and are well worth your time and study.



Complete the table using the triangles:

Recall: SOH-CAH-TOA

θ	30°	45°	60°
Sin θ			
Cos θ			
Tan θ			

Yes

But, use the triangles to help organize the details

Example #1: If $h(x) = 10\cos x + 3$, find the value of $h(45^\circ)$.

Example #2: If $f(x) = \sin x$ and $g(x) = 2x$, find $f(g(30^\circ))$.

Example #3: Given: $f(x) = \tan x$ and $g(x) = \frac{1}{2}x$. Evaluate: $f(g(60^\circ))$.

Using Reference Angles:

When our angle moves **into quadrants II, III, or IV** we must be aware of the sign of our Trigonometric function value. We will use the reference angles to help us. There are two types of questions we can see where we need to use this process:

- 1) Writing as a function of a positive acute angle
- 2) Finding the exact value when the angles are greater than 90° .

QSFR:

To begin we will be writing these as functions of a positive acute angle by filling in the boxes:

Example: Write the following as a function of a positive acute angle.

$$\sin 250^\circ$$

Quadrant	Sign	Function	Reference
Q	S	F	R

Example #4: Express the following as a function of a positive acute angle.

(a) $\cos 170^\circ$

Quadrant	Sign	Function	Reference
Q	S	F	R

(b) $\tan(-340^\circ)$

Quadrant	Sign	Function	Reference
Q	S	F	R

(c) $\sin 580^\circ$

Quadrant	Sign	Function	Reference
Q	S	F	R

(d) $\cos(-140^\circ)$

Quadrant	Sign	Function	Reference
Q	S	F	R

Example #5: Find the **EXACT** value

(a) $\cos 300^\circ$

Quadrant	Sign	Function	Reference
Q	S	F	R

(b) $\cos 570^\circ$

Quadrant	Sign	Function	Reference
Q	S	F	R

(c) $\sin 240^\circ$

Quadrant	Sign	Function	Reference
Q	S	F	R

(d) $\tan(-135^\circ)$

Quadrant	Sign	Function	Reference
Q	S	F	R

Sometimes, we must use this concept, to find the coordinates of angles on the unit circle. The same concept is applied.

Example #6: Draw a rotation diagram for each of the following angles and then determine the ordered pair that lies on the unit circle.

(a) 330°

(b) 135°

(c) -270°

(d) 540°

(e) 675°

(f) -120°

Example #7: Which of the following is *not* equivalent to $\tan 498^\circ$?

(1) $\cos 138^\circ$

(2) $-\tan 42^\circ$

(3) $\tan 48^\circ$

(4) $\tan 858^\circ$

Example #8: The expression $\sin 240^\circ$ is equivalent to

(1) $\sin 60^\circ$

(2) $\cos 60^\circ$

(3) $-\sin 60^\circ$

(4) $-\cos 60^\circ$

Chapter 11A: Trigonometry Unit 1
Lesson 5: Trig Exact Values
Homework

1.) Written in exact form, $\cos(135^\circ) = ?$

(1) $-\frac{1}{2}$

(2) $-\frac{\sqrt{2}}{2}$

(3) $-\frac{\sqrt{3}}{2}$

(4) $-\frac{\pi}{4}$

2.) Which of the following is not equal to $\sin(270^\circ)$?

(1) $\cos 180^\circ$

(2) $-\cos 0^\circ$

(3) $-\sin 90^\circ$

(4) $\sin 360^\circ$

3.) If $f(x) = 10 \sin x - 3$ then $f(30^\circ) = ?$

(1) $-\frac{\sqrt{3}}{2} - 3$

(2) 2

(3) $-\frac{5}{2}$

(4) $\frac{4}{3} - \frac{\sqrt{3}}{2}$

4.) Which of the following is equal to $\sin(300^\circ)$?

(1) $\sin(60^\circ)$

(2) $\sin(30^\circ)$

(3) $-\sin(60^\circ)$

(4) $-\sin(30^\circ)$

5.) Find the exact value of the functions below.

(a) $\sin 330^\circ + \cos 120^\circ$

(b) $\sin 300^\circ + \cos 150^\circ$

(c) $\tan 210^\circ + \sin 330^\circ$

(d) $\cos 90^\circ - \tan 135^\circ$

6.) Expressed as a function of a positive acute angle, $\cos(-305^\circ)$ is equal to

(1) $-\cos 55^\circ$

(2) $\cos 55^\circ$

(3) $-\sin 55^\circ$

(4) $\sin 55^\circ$

Chapter 11A: Trigonometry Unit 1
Lesson 6: Trig Exact Values Radian Measures

Steps:

- 1.) Change from radians into degrees.
- 2.) Find the positive acute angle the same way we did by using QSFR.
- 3.) Change your angle back into radians.

Example 1: Rewrite each function as a function of a positive acute angle.

(a) $\sin \frac{7\pi}{12}$

(b) $\tan \frac{16\pi}{9}$

(c) $\cos \frac{13\pi}{9}$

(d) $\cos \frac{19\pi}{12}$

Example 2: Find the *exact* value of the following functions.

(a) $\tan \frac{7\pi}{6}$

(b) $\cos \frac{3\pi}{4}$

(c) $\sin \frac{5\pi}{6} + \cos \frac{3\pi}{2}$

(d) $\cos \frac{5\pi}{4} + \tan \frac{3\pi}{4}$

Example 3: Draw a rotation diagram for each of the following radian angles and then determine the ordered pair that lies on the unit circle for each angle.

(a) $\theta = \frac{2\pi}{3}$

(b) $\theta = -\frac{3\pi}{2}$

$$(c) \theta = \frac{11\pi}{6}$$

$$(d) \theta = \frac{4\pi}{3}$$

Example 4: If $f(x) = \cos x + \tan \frac{x}{3}$, then $f(\pi)$ is

$$(1) \frac{\sqrt{3}+3}{3}$$

$$(2) \frac{\sqrt{3}-3}{3}$$

$$(3) \sqrt{3} + 1$$

$$(4) \sqrt{3} - 1$$

Example 5: At $x = \frac{\pi}{2}$, the difference $2 \sin x - \cos 2x$ is

$$(1) 1$$

$$(2) 2$$

$$(3) 3$$

$$(4) 0$$

Chapter 11A: Trigonometry Unit 1
Lesson 6: Trig Exact Values Radian Measures
Homework

1.) Which of the following represents a rational number?

(1) $\sin \frac{\pi}{6}$

(2) $\sin \frac{2\pi}{3}$

(3) $\cos \frac{\pi}{4}$

(4) $\cos \frac{5\pi}{4}$

2.) If $f(x) = 2x$ and $g(x) = \cos x$, then $g\left(f\left(\frac{\pi}{2}\right)\right) = ?$

(1) 1

(2) $-\frac{\sqrt{2}}{2}$

(3) 0

(4) -1

3.) If $f(x) = 4 \cos 3x$, what is the value of $f\left(\frac{\pi}{4}\right)$?

(1) $-\sqrt{2}$

(2) $-2\sqrt{2}$

(3) 135

(4) 4

4.) Find the value of each of the following trig functions. Your answer must be **exact**.

(a) $\sin \frac{5\pi}{3}$

(b) $\cos \left(-\frac{2\pi}{3}\right)$

(c) $\tan \left(\frac{2\pi}{3}\right)$

(d) $\tan \left(\frac{11\pi}{6}\right)$

5.) Draw a rotation diagram for each of the following radian angles and then determine the ordered pair that lies on the unit circle for each angle.

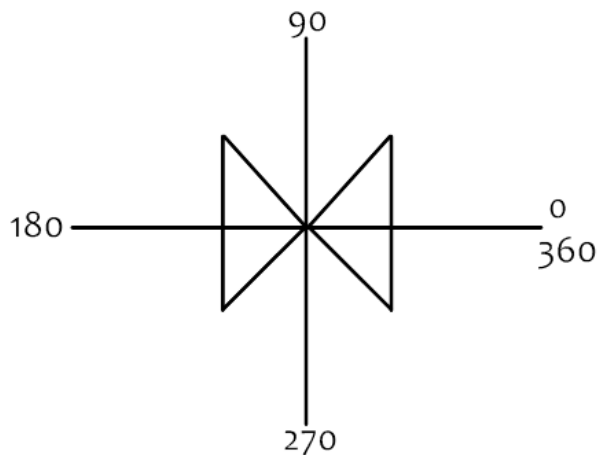
(a) $\alpha = \frac{3\pi}{4}$

(b) $\alpha = \frac{7\pi}{6}$

Chapter 11A: Trigonometry Unit 1
Lesson 7: Unit Circle “Bow Tie” Problems

“Bow Tie” Problems:

Sometimes, when dealing with trig, we are asked to determine the sin, cos, or tan of an angle without being given the specific angle measure. To do this, we must draw out triangles in the quadrants and use SOH-CAH-TOA & Pythagorean Theorem to solve.



1.) If the value of $\sin\theta = 12/13$ and $\cos\theta > 0$, what is the value of $\tan\theta$?

- Steps:**
- 1.) Find which quadrant the angle lies in.
 - 2.) Determine if trig function is + or -.
 - 3.) Draw a triangle in the quadrant.
 - 4.) Use Pythagorean Theorem to find missing side.
 - 5.) Use SOH-CAH-TOA.

2.) If the value of $\tan\theta = -10/9$ and $\sin\theta > 0$, find the value of $\cos\theta$. Your answer must be in simplest radical form.

3.) The value of $\cos\theta = -6/7$ and $\tan\theta < 0$, find the value of $\sin\theta$. Final answer must be in simplest radical form.

4.) If $\cos\theta = -4/5$ and θ lies in Quadrant II, what is the value of $\tan\theta$?

(1) $3/4$

(2) $4/3$

(3) $-3/4$

(4) $-4/3$

5.) If $\cos\theta = 9/41$ and θ lies in Quadrant IV, what is the value of $(\sin\theta)(\tan\theta)$?

6.) If $\sin\theta = -2/5$ and θ lies in Quadrant III, what is the value of $(\cos\theta)(\tan\theta)$? Final answer must be in simplest radical form.

7.) If $\tan\theta = 4/3$ and θ lies in Quadrant III, what is the value of $(\sin\theta) + (\cos\theta)$?

8.) If $\cos\theta = -1/5$ and θ lies in Quadrant II, what is the value of $(\tan\theta) + (\sin\theta)$? Final answer must be written as a single fraction in simplest radical form.

9.) When drawn in standard position, an angle α has a terminal ray that lies in the second quadrant and whose sine is equal to $9/41$. Find the cosine of α in rational form (as a fraction).

Chapter 11A: Trigonometry Unit 1
Lesson 7: Unit Circle "Bow Tie" Problems
Homework

1.) If the value of $\sin \theta = \frac{8}{17}$ and $\tan \theta < 0$, what is the value of $\cos \theta$?

2.) If the value of $\tan \theta = -\frac{15}{7}$ and $\cos \theta < 0$, what is the value of $\sin \theta$?

3.) If $\sin \theta = \frac{2}{3}$ and θ is in Quadrant II, what is the value of $(\tan \theta)(\cos \theta)$? Answer must be in simplest radical form.

4.) $\tan \theta = -\frac{5}{7}$ and θ is in Quadrant IV, what is the value of $\sin \theta + \cos \theta$?

5.) Determine the value of $\cos x$ and $\tan x$ if $\sin x = \frac{5}{13}$ and the terminal ray of x lies in the second quadrant.

6.) The point $A(-5,12)$ lies on the circle whose equation is $x^2 + y^2 = 169$. Which of the following would represent the cosine of an angle drawn in standard position whose terminal rays passes through A ?

(1) -5

(2) $-\frac{5}{12}$

(3) $-\frac{5}{13}$

(4) 12

7.) If the terminal ray of β lies in the fourth quadrant and $\sin \beta = -\frac{\sqrt{3}}{3}$ determine $\cos \beta$ in simplest form.