Lesson 3
Completing the Square and Shifting Parabolas

Parabolas, and graphs more generally, can be moved horizontally and vertically by simple manipulations of their equations. This is known as **shifting** or **translating** a graph. The first exercise will review how to use a method known as **completing the square** to identify shifts and the turning point of a parabola.

Completing the square is a useful tool that can be used to **modify the appearance of an equation**. With quadratic equations, this can be used to **solve for the roots of the equation**, or to **identify the vertex** of the equation's parabola.

Let's review the steps from Algebra I:

Identify all solutions by completing the square: $x^2 - 20 = -8x$

- Put equation in standard form and make the leading coefficient ONE.
- Move the constant to the other side & make spaces.
- Write down your key number $\left(\frac{b}{2}\right)$ and add the square to both sides.
- Factor the perfect left side using the Key
- Take the square root of both sides. *Use the $\pm$ on the right side!*
- Solve

Identify the vertex of the parabola of $y = x^2 + 6x + 1$

- Put equation in standard form and make the leading coefficient ONE.
- Ignoring the y, move the constant to the other side & make spaces.
- Write down your key number $\left(\frac{b}{2}\right)$, and add the square to both sides.
- Factor the perfect left side using the Key
- Bring the constant back over & write as "y=" again
- Identify the vertex (h, k)
    - x switches! y stays the same!

***Exercise* #1:** Place each of the following quadratic functions in **vertex form** and identify the turning **point**.

Remember! For completing the square the leading coefficient MUST be 1! If it's not to begin with, divide it away!

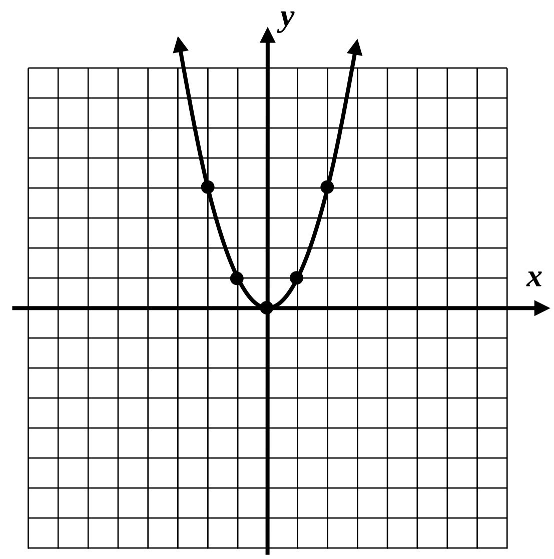
(a) $y = x^2 - 12x + 11$

(b) $y = 6x^2 + 12x - 48$

(c) $y = 3x^2 + 12x - 2$

(d) $y = 2x^2 + 6x + 1$

***Exercise* #2:** The function $y = x^2$ is shown already graphed on the grid below. Consider the quadratic whose equation is $y = x^2 - 8x + 18$.



(a) Using the method of completing the square, write this equation in the form $y = (x-h)^2 + k$. Identify the vertex.

(b) Sketch the graph of $y = x^2 - 8x + 18$ by using its **vertex form** in (a).

(c) Describe how the graph of $y = x^2$ would be shifted to produce the graph of the equation found in part (b)

Summary of Graph Shifts of Parabolas.

Based on the parent function of $\mathbf{y = x^2}$, the graph of $\mathbf{y = (x - h)^2 + k}$ shifts

h: Horizontal Translation. Moves the graph **left** or **right**

$(\boldsymbol{x} + \boldsymbol{h})$ moves left $(\boldsymbol{x} - \boldsymbol{h})$ moves right

k: Vertical Translation. Moves the graph **up** or **down**

$+\boldsymbol{k}$ moves up $-\boldsymbol{k}$ moves down

Vertex of the parabola: (h, k)

The method of completing the square can be performed on the standard quadratic equation $y = ax^2 + bx + c$ and after much manipulation, the x-value of the vertex can be expressed as $\boldsymbol{x = -\frac{b}{2a}}$. Then, to find the y-value of the vertex, we plug that x-value back into the function.

$$\text{Vertex} = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$

***Exercise* #3:** Use the formula $x = -\frac{b}{2a}$ to find the turning **point** for each of the following quadratic functions. Then, graph each on your calculator to verify your answer

(a) $f(x) = 2x^2 - 12x + 7$

(b) $g(x) = -\frac{1}{4}x^2 + 5x - 20$

Summary of ways to identify the vertex (turning point) of a quadratic equation:

- Put the equation in VERTEX FORM $\boldsymbol{y = (x - h)^2 + k}$ by completing the square. **Vertex =** $\boldsymbol{(h, k)}$
- In STANDARD FORM $\boldsymbol{y = ax^2 + bx + c}$, calculate the vertex using a, b, c values. **Vertex =** $\boldsymbol{\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)}$
- Utilize your calculator's table, graph, or minimum/maximum features to identify the vertex.

LESSON 3 HOMEWORK
COMPLETING THE SQUARE AND SHIFTING PARABOLAS

FLUENCY

1. Which of the following equations would result from shifting $y=x^2$ five units right and four units up?

(1) $y=(x-5)^2+4$ (3) $y=(x-4)^2-5$

(2) $y=(x+5)^2+4$ (4) $y=(x+4)^2-5$

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2. Which of the following represents the turning point of the parabola whose equation is $y=(x+3)^2-7$?

(1) $(3,-7)$ (3) $(-7,-3)$

(2) $(-3,7)$ (4) $(-3,-7)$

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3. Which of the following quadratic functions would have a turning point at $(6,-2)$?

(1) $y=(x+6)^2-2$ (3) $y=5(x-6)^2-2$

(2) $y=3(x+2)^2-2$ (4) $y=2(x-1)^2+6$

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4. Which of the following is turning point of $y=x^2+12x-4$?

(1) $(12,-4)$ (3) $(6,104)$

(2) $(-6,-40)$ (4) $(-4,12)$

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5. In vertex form, the parabola $y=x^2-10x+8$ would be written as

(1) $y=(x-5)^2-33$ (3) $y=(x-10)^2-92$

(2) $y=(x-5)^2-17$ (4) $y=(x-10)^2-108$

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6. The turning point of the parabola $y=x^2+5x-2$ is

(1) $(2.5,12.75)$ (3) $(-2.5,-8.25)$

(2) $(-5,-10.5)$ (4) $(-2.5,-17.5)$

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7. Write each of the following quadratic functions in its vertex form by completing the square. Then, identify its turning point.

(a) $y = x^2 + 12x + 50$

(b) $y = -3x^2 + 30x + 7$

8. Use the formula $x = -\frac{b}{2a}$ to find the turning points of each of the following quadratic functions. Then, place the function in vertex form to verify the turning points.

(a) $y = 5x^2 - 30x + 55$

(b) $y = -2x^2 - 24x - 67$

9. Consider the quadratic function whose equation is $y = x^2 + 6x - 40$.

(a) Determine the y-intercept of this function algebraically.

(b) Write the function in its vertex form. State the coordinates of its turning point.

(c) Algebraically find the zeroes of the function using the zero product law.

(d) Sketch a graph of the parabola, showing all relevant features found in parts (a) through (c).

10. Using the ‘Completing the Square’ method, put the standard form of a quadratic equation into vertex form. Then, identify the vertex.

$$\boldsymbol{ax^2 + bx + c = 0}$$