

**Chapter 5: Quadratics**  
**Topic 7: Sum and Product of the Root**

**Sum & Product of the Roots - General**

**Recall:** The ROOTS of an equation are what we get when the equation is set equal to zero and we solve. Also known as the x-intercepts, the zeros, or the solutions.

We will sometimes be asked to find the sum of the roots and/or the product of the roots.

- To find the sum of the roots, we could solve the equation and add together the two solutions.
- To find the product of the roots, we could solve the equation and multiply the two solutions.

OR... We can take a shortcut...

**Given the standard form:**

**Sum of the Roots =**

**Product of the Roots =**

**Examples:** Identify a, b, c. Write the formula. Plug in and reduce.

1.  $3x^2 - 6x + 11 = 0$

2.  $x^2 + 8x = -7$

Prove it to yourself! For question #2, factor, solve & find sum and product by hand. Are they the same?

**Multiple Choice Practice:**

3. What are the sum(s) and product (p) of the roots the equation  $x^2 + 3x - 27 = 0$ .

(1)  $s = 3, p = 27$       (3)  $s = -27, p = 2$

(2)  $s = -3, p = -27$     (4)  $s = -27, p = -3$

4. What is the product of the roots of the equation  $-8x^2 + 3x + 2 = 0$ ?

(1)  $-\frac{1}{4}$                       (2) 2

(3)  $\frac{3}{16}$                         (4) -2

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**Standard equation modified for Sum & Product:**

$$x^2 - (\text{sum})x + (\text{product}) = 0$$

Two very important things to notice:

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**Write an Equation: Given the SUM & PRODUCT**

Two options could happen...

**If you're given integers, just plug in:**

5. The sum of the roots of a quadratic equation is 12 and the product is  $-4$ . Write a quadratic equation.

**If you're given fractions, get an LCD, plug in, and multiply to clear the denominators:**

6. Write a quadratic equation, with integral coefficients whose roots have the following sum and products:

$$\text{sum} = \frac{-3}{4} \quad \text{product} = \frac{-1}{2}$$

**You try**, in your notebook: Write a quadratic equation whose roots have the following sum and product. All equations must have integral coefficients

7.  $\text{sum} = 4, \text{product} = -9$

8.  $\text{sum} = \frac{4}{5}, \text{product} = \frac{2}{3}$

9.  $\text{sum} = \frac{-1}{6}, \text{product} = \frac{1}{3}$

10.  $\text{sum} = -3, \text{product} = \frac{4}{5}$

**Write an Equation: Given the ROOTS**

Take the original sum & product equation:

$$x^2 - \text{sum } x + \text{product} = 0$$

Think about what sum (addition) and product (multiplication) mean.

If we call the two roots " $r_1$ " and " $r_2$ ", then the sum is  $r_1 + r_2$ , and the product is  $r_1 \cdot r_2$ . We can rewrite the equation as:

Examples:

11. Write a quadratic formula whose roots are  $-3$  and  $7$ .

- Write the formula
- Calculate sum
- Calculate product
- Plug in & simplify

12. Write a quadratic equation with integral coefficients whose roots are  $\frac{3}{4}$  and  $\frac{1}{2}$ .

- Write the formula
- Calculate sum
- Calculate product
- Rewrite with LCD
- Plug in & simplify

**You try**, in your notebook. Write the quadratic equation with integral coefficients which have the following roots:

13. Roots:  $\frac{2}{5}$  and  $\frac{4}{3}$

14. Roots:  $\frac{2}{3}$  and  $\frac{5}{6}$

15. Roots:  $(3 + \sqrt{5})$  and  $(3 - \sqrt{5})$

16. Roots:  $(2 + 3\sqrt{2})$  and  $(2 - 3\sqrt{2})$

17. Roots:  $(3 + 4i)$  and  $(3 - 4i)$

18. One root of  $(5 + 6i)$

19. One root of  $4 + \sqrt{7}$

**Hint for 18 & 19:**

Look at the previous examples... what is the relationship between the roots when they are irrational or imaginary?

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Use that to determine the second root!

**Given an incomplete equation and one root... find the missing value and second root****Example:****20.** If a root of the equation  $x^2 - 6x + k = 0$  is 4, find the second root and the missing value.*Collect the pieces that we know:*

$$a = \quad b = \quad c =$$

*Plug in as much as we can...*SUM:

$$-\frac{b}{a} = r_1 + r_2$$

PRODUCT:

$$\frac{c}{a} = r_1 \cdot r_2$$

**Final answer:**  $k =$                    $r_2 =$ **21.** Given the equation  $x^2 + kx + 18 = 0$  with one root of 6, find the second root and the missing value.*Collect the pieces that we know:*

$$a = \quad b = \quad c =$$

*Plug in as much as we can...*SUM:

$$-\frac{b}{a} = r_1 + r_2$$

PRODUCT:

$$\frac{c}{a} = r_1 \cdot r_2$$

**Final answer:**  $k =$                    $r_2 =$ **You try**, in your notebook. For these equations, one root is given. Find the second root and the missing value.

**22.**  $x^2 - x + k = 0$        $r_1 = -4$

**23.**  $2x^2 + bx - 15 = 0$        $r_1 = 3$

**24.**  $3x^2 - x + k = 0$        $r_1 = \frac{-5}{3}$