

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

**Chapter 3: Exponential & Logarithmic Functions**  
**Topic 5: Modeling with Exponential & Log Functions**

**Exponential Growth & Decay Model**  $A = P(e)^{rt}$

*In these questions, other pieces may be missing instead of just plugging in!*

**Example:** The graph shows the growth of the minimum wage from 1970 through 2000.

a. Find the exponential growth function that models the data for 1970 through 2000.

- Plug in what you know
- Use any known point to solve for  $r$



b. By which year will the minimum wage reach \$7.50 per hour?

**Example:** The half life of Carbon-14 is 5715 years. That is, after 5715 years, a sample of Carbon-14 will have decayed to half of the amount. What is the exponential decay model for Carbon-14?

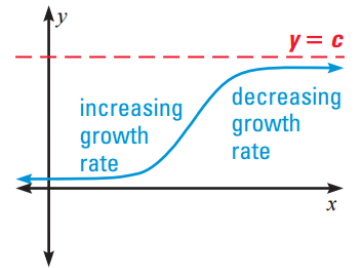
- Begin with the exponential decay model.
- In terms of  $P$ , what will the output ( $A$ ) be?
- Divide both sides by  $P$
- Log to solve for  $k$

## Logistic Growth Model

$$A = \frac{c}{1+a(e)^{-bt}}$$

Where  $a$ ,  $b$  and  $c$  are constants with  $c > 0$  and  $b > 0$

Unlike exponential growth which has no upper bound (can increase infinitely), logistic growth does have an upper bound. As time increases, the expression  $a(e)^{-bt}$  approaches zero, making  $A$  approach  $c$ . Therefore,  **$c$  is the limit to the growth of  $A$**  and creates a horizontal asymptote for the graph of these functions



**Example:** The function  $f(t) = \frac{30000}{1+20e^{-1.5t}}$  describes the number of people who have become ill with influenza  $t$  weeks after its initial outbreak in a town with 30,000 inhabitants.

How many people became ill with the flu when the epidemic began?

How many people were ill by the end of the 4<sup>th</sup> week?

What is the limited size of the population that becomes ill?

## Newton's Law of Cooling

The temperature,  $T$ , of a heated object at time  $t$  is given by  $T = C + (T_0 - C)e^{kt}$

Where:  $C$  is the constant temperature of the surrounding medium (often room temperature).

$T_0$  is the initial temperature of the heated object.

$k$  is a negative constant that is associated with cooling object.

Example: A cake removed from the oven has a temperature of 210°F. It is left to cool in a room that has a temperature of 70°F. After 30 minutes, the temperature of the cake is 140°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the cake,  $T$ , after  $t$  minutes.

b. What is the temperature of the cake after 40 minutes?

c. When will the temperatures of the cake be 90°F?

## Formulas you need to know from this chapter:

Compound Interest	$A = P \left(1 + \frac{r}{n}\right)^{nt}$
Continuous Compounding Exponential Growth/Decay	$A = P(e)^{rt}$
Logistic Growth	$A = \frac{c}{1 + a(e)^{-bt}}$
Newton's Law of Cooling	$T = C + (T_0 - C)e^{kt}$

